

Michael Schulz
RA 5a MNG V

Michael Schulz Rechnen - Algebra V Sa MNG

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1)
$$\begin{array}{r} 210_{(3)} \\ + 122_{(3)} \\ \hline 1102_{(3)} \end{array} \checkmark$$

2)
$$\begin{array}{r} 2101_{(3)} \\ - 211_{(3)} \\ \hline 1120_{(3)} \end{array} \checkmark$$

3) $1001_{(3)} : 11_{(3)} = 21_{(3)}$

$$\begin{array}{r} 22 \\ 11 \\ \hline 11 \\ 0 \end{array}$$

4)
$$\underline{\underline{66666}}_{(7)} \checkmark$$

5) a) $1000 \Rightarrow 1101001_{(3)} \checkmark$ b) $1000 \Rightarrow 13000_{(5)} \checkmark$

| | |
|---|--------|
| 0 | 333 R1 |
| 0 | 111 R0 |
| 0 | 37 R0 |
| 1 | 12 R1 |
| 1 | 4 0 |

c) $1000 \begin{matrix} 166 R4 \\ 27 R4 \\ 4 R3 \\ R4 \end{matrix} \Rightarrow \underline{\underline{4344}}_{(6)} \checkmark$ d) $1000 \begin{matrix} 142 R6 \\ 20 R2 \\ 2 R6 \\ R2 \end{matrix} \Rightarrow \underline{\underline{2626}}_{(7)} \checkmark$

e) $1000 \begin{matrix} 125 R0 \\ 15 R5 \\ 1 R7 \\ 0 R1 \end{matrix} \Rightarrow \underline{\underline{1750}}_{(8)} \checkmark$ f) $1000 \begin{matrix} 111 R1 \\ 12 R3 \\ 1 R3 \\ 0 R1 \end{matrix} \Rightarrow \underline{\underline{1331}}_{(9)} \checkmark$

5093 $\begin{matrix} 1697 R2 \\ 565 R2 \\ 188 R1 \\ 62 R2 \\ 20 R2 \\ 6 R2 \\ 2 R0 \\ 0 R2 \end{matrix} \Rightarrow \underline{\underline{20222122}}_{(3)} \checkmark$

5093 $\begin{matrix} 1018 R3 \\ 203 R3 \\ 40 R3 \\ 13 R0 \\ 4 R1 R3 \\ 1 R1 R1 \\ 0 R1 \end{matrix} \Rightarrow \underline{\underline{111333}}_{(5)} \checkmark$

5093 $\begin{matrix} 848 R5 \\ 141 R2 \\ 23 R3 \\ 3 R5 \\ 0 R3 \end{matrix} \Rightarrow \underline{\underline{35325}}_{(6)} \checkmark$

5093 $\begin{matrix} 727 R4 \\ 103 R6 \\ 14 R5 \\ 2 R0 \\ 0 R2 \end{matrix} \Rightarrow \underline{\underline{20564}}_{(7)} \checkmark$

5093 $\begin{matrix} 636 R5 \\ 79 R4 \\ 9 R7 \\ 1 R1 \\ 0 R1 \end{matrix} \Rightarrow \underline{\underline{11745}}_{(8)} \checkmark$

5093 $\begin{matrix} 565 R8 \\ 62 R7 \\ 6 R8 \\ 0 R6 \end{matrix} \Rightarrow \underline{\underline{6878}}_{(9)} \checkmark$

$$c) 20012_{(3)} = 2 + 3 + 162 = \underline{167} \checkmark$$

$$444444_{(5)} = 4 + 20 = \underline{78124} \checkmark$$

$$901_{(11)} = 1 + 1089 = \underline{1090} \checkmark$$

$$1053_{(6)} = \underline{249} \checkmark$$

$$7a) 303_{(4)} = 51 = 25 R1 \Rightarrow \underline{110011}_{(2)} \checkmark$$

12 R1
6 R0
3 R0
1 R1
R1

$$303_{(4)} : 2_{(4)} = 116 R1$$

$$111_{(4)} : 2_{(4)} = 22 R1$$

$$22_{(4)} : 2_{(4)} = 11 R0$$

$$11_{(4)} : 2_{(4)} = 2 R1$$

$$b) 2332_{(4)} = 190 = 95 R0 \Rightarrow 1011110_{(2)} \checkmark$$

47 R1
23 R1
11 R1
5 R1
2 R1
1 R0
R1

$$8) a) 210_{(3)} = 21 = 3 R0 \Rightarrow \underline{30}_{(7)} \checkmark$$

$$b) 2222_{(3)} = 80 = 11 R3 \Rightarrow \underline{143}_{(7)} \checkmark$$

1 R4
R1

$$9) 206_{(7)} \cdot 453_{(7)}$$

$$\begin{array}{r} 4044 \\ 123600 \\ \hline 130644_{(7)} \checkmark \end{array}$$

$$10) 4310_{(5)} \cdot 24321_{(5)}$$

$$\begin{array}{r} 243210 \\ 13401300 \\ 213334000 \\ \hline 233034010_{(5)} \checkmark \end{array}$$

$$11) 61235_{(7)}$$

$$\begin{array}{r} -6666_{(7)} \\ \hline 51236_{(7)} \checkmark \end{array}$$

$$12) 23587_{(9)}$$

$$\begin{array}{r} +71468_{(9)} \\ \hline 105166_{(9)} \checkmark \end{array}$$

$$13) 67_{(9)} \cdot 184_{(9)}$$

$$\begin{array}{r} 1451 \\ 12560 \\ \hline 14121_{(9)} \checkmark \end{array}$$

$$14) 46787_{(9)} : 8_{(9)} = 5322_{(9)} \checkmark$$

$$\begin{array}{r} 27 \\ 18 \\ 17 \end{array}$$

$$15) 63214_{(7)} : 4_{(7)} = \underline{1422} \checkmark$$

$$\begin{array}{r} 23 \\ 12 \\ 11 \end{array}$$

$$17) 2154_{(6)} \cdot 513204_{(6)}$$

$$\begin{array}{r} 3301224 \\ 42144320 \\ 51320400 \\ 1430412000 \\ \hline 2022022344_{(6)} \checkmark \end{array}$$

$$16) 1243134_{(5)}$$

$$\begin{array}{r} 3110444_{(5)} \\ 324012_{(5)} \\ 141223_{(5)} \\ 2324411_{(5)} \\ \hline 10004334_{(5)} \checkmark \end{array}$$

$$18) 46302_{(7)} \cdot 666_{(7)} =$$

$$\begin{array}{r} 666_{(7)} \cdot 46302_{(7)} \\ \hline \end{array}$$

$$\begin{array}{r} 413415 \\ 4134150 \\ 41341500 \\ \hline \end{array}$$

$$\underline{\underline{46222365_{(7)}}} \checkmark$$

$$19) 1244021_{(5)} =$$

$$\begin{array}{r} 1244021_{(5)} \\ - 440242_{(5)} \\ \hline \end{array}$$

$$\underline{\underline{303224_{(5)}}} \checkmark$$

$$20) 8888_{(9)} \cdot 8888_{(9)} =$$

$$\begin{array}{r} 8888_{(9)} \cdot 8888_{(9)} \\ \hline 78887 \\ 7888810 \\ 7888100 \\ 788821000 \\ \hline \end{array}$$

$$\underline{\underline{88870001_{(9)}}} \checkmark$$

Systembrüche:

$$948.143 = 9 \cdot 10^2 + 4 \cdot 10^1 + 8 \cdot 10^0 + 1 \cdot \frac{1}{10^1} + 4 \cdot \frac{1}{10^2} + 3 \cdot \frac{1}{10^3}$$

$$31.213_{(4)} = 3 \cdot 4^1 + 1 \cdot 4^0 + 2 \cdot \frac{1}{4^1} + 1 \cdot \frac{1}{4^2} + 3 \cdot \frac{1}{4^3}$$

$$110001,100011_{(2)} = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^1 + 1 \cdot \frac{1}{2^1} + 1 \cdot \frac{1}{2^5} + 1 \cdot \frac{1}{2^6}$$

$$\text{allg.: } abc,de_{(n)} = a \cdot n^2 + b \cdot n^1 + c \cdot n^0 + d \cdot \frac{1}{n^1} + e \cdot \frac{1}{n^2}$$

$$\frac{1}{2^3} + \frac{1}{2} = 0.101_{(2)}$$

$$\frac{4}{7^3} + \frac{2}{7^2} + \frac{5}{7} = 0.524_{(7)}$$

$$abc_{(n)} : 100_{(n)} = a.b c_{(n)}$$

$$23_{(5)} : 100_{(5)} = 0.23_{(5)}$$

Potenzen mit ganzen Exponenten

Def. $a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n\text{-Faktoren}}$

a: Basis
n: Exponent

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$$

Regeln: 1. Die Potenzen der positiven, reellen Zahlen sind positiv.

2. $a \in \mathbb{R}, a < 0$

$$a^{2n} > 0$$

$$a^{2n+1} < 0$$

$$\begin{aligned} (-x)^{\text{gerade Potenz}} &= > 0 \\ -(-x)^{\text{gerade Potenz}} &= < 0 \end{aligned}$$

Potenz eines Produktes

$$\underbrace{(a \cdot b)^n}_{n\text{-Faktoren}} = \underbrace{(a \cdot b)(a \cdot b) \dots (a \cdot b)}_{n\text{-Faktoren}} = \underbrace{a \cdot a \cdot a \dots a}_{n\text{-Faktoren}} \cdot \underbrace{b \cdot b \cdot b \dots b}_{n\text{-Faktoren}} = a^n b^n$$

Kommutativgesetz (KG)
der Multiplikation

Bsp.: $(2z)^3 = 2^3 \cdot z^3 = 8z^3$

$$32a^5 = (2a)^5$$

analog für drei Faktoren:

$$(a \cdot b \cdot c)^n \stackrel{\text{KG}}{=} a \cdot b \cdot c \cdot a \cdot b \cdot c \cdot a \cdot b \cdot c \dots a \cdot b \cdot c = a^n \cdot b^n \cdot c^n$$

für endlich viele Faktoren

$$(a \cdot b \cdot c \cdot d \dots e)^n = a^n b^n c^n d^n \dots e^n$$

$$(2xy)^5 = 32x^5y^5$$

$$(64z^3y^3)^3 = (4zy)^3$$

$$8(a^4b^4) = (3ab)^4$$

Potenz eines Quotienten

$$\left(\frac{a}{b}\right)^n = \frac{a}{b} \cdot \frac{a}{b} \cdot \dots \cdot \frac{a}{b} = \frac{\overbrace{a \cdot a \cdot a \cdot \dots \cdot a}^{n\text{-Faktoren}}}{\underbrace{b \cdot b \cdot b \cdot \dots \cdot b}_{n\text{-Faktoren}}} = \frac{a^n}{b^n}$$

Bsp.: 1) $\left(\frac{1}{c}\right)^k = \frac{1}{c^k}$

2) $\left(\frac{0,1cx}{rs}\right)^4 = \frac{(0,1cx)^4}{(rs)^4} = \frac{0,0001c^4x^4}{r^4s^4}$

3) $\frac{48^2}{16^2} = \left(\frac{48}{16}\right)^2 = 3^2 = 9$

5/2 a) $2n = n+n$

$$n^2 = n \cdot n$$

$$2^n = \underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{n\text{-Faktoren}}$$

c) $m \cdot \underbrace{a \cdot \dots \cdot a}_{n\text{-Faktoren}}$

$$m \cdot \dots \cdot m \cdot a \cdot \dots \cdot a$$

d) $(a+b)(a+b) \cdot \dots \cdot (a+b)$

$$a \cdot \dots \cdot a + b \cdot \dots \cdot b$$

6/9 Berechne die Quadratzahlen von 21^2 bis 30^2 , indem du die Formeln $(a+b)^2$ und $(a-b)^2$ verwendest.

$$(21)^2 = ((20+1)(20+1))^2 = 441^2 = ((20+1)(22-1)) = 440 - 20 + 22 - 1 = 441$$

$$22^2 = (20+2)(20+2) = 400 + 40 + 40 + 4 = 484$$

$$(25-3)(19+3) = 475 + 75 - 57 - 9 = 484$$

$$23^2 = (20+3)(20+3) = 400 + 60 + 60 + 9 = 529$$

$$(18+5)(28-5) = 504 - 90 + 140 - 25 = 529$$

$$24^2 = (10+14)(10+14) = 100 + 140 + 140 + 196 = 576$$

$$(22+2)(26-2) = 572 + 52 - 44 - 4 = 576$$

$$(25-1)(25-1) = 625 - 25 - 25 + 1 = 576$$

$$25^2 = (20+5)(20+5) = 400 + 100 + 100 + 25 = 625$$

$$(30-5)(30-5) = 900 - 150 - 150 + 25 = 625$$

$$26^2 = (25+1)(25+1) = 625 + 25 + 25 + 1 = 676$$

$$(30-4)(30-4) = 900 - 120 - 120 + 16 = 676$$

$$27^2 = (20+7)(20+7) = 400 + 140 + 140 + 49 = 729$$

$$(30-3)(30-3) = 900 - 90 - 90 + 9 = 729$$

$$28^2 = (20+8)(20+8) = 400 + 160 + 160 + 64 = 784$$

$$(30-2)(30-2) = 900 - 60 - 60 + 4 = 784$$

$$29^2 = (20+9)(20+9) = 400 + 180 + 180 + 81 = 841$$

$$(30-1)(30-1) = 900 - 30 - 30 + 1 = 841$$

$$30^2 = (20+10)(20+10) = 400 + 200 + 200 + 100 = 900$$

$$(40-10)(40-10) = 1600 - 400 - 400 + 100 = 900$$

Multiplikation von Potenzen mit gleicher Potenzen Basis

$$10^4 \cdot 10^3 = 10^7 = 10^{4+3}$$

$$\underline{a^m \cdot a^p} = \underbrace{a \cdots a}_m \cdot \underbrace{a \cdots a}_p = a^{m+p}$$

m-Faktoren p-Faktoren

$$a^m \cdot a^p \cdot a^q = a^{m+p} \cdot a^q = a^{m+p+q}$$

$$\text{Bsp.: } 2^{n+1} \cdot 2^{n-1} = 2^{2n}$$

$$5^{3n-1} \cdot 5^{7n+4} = 5^{10n+3}$$

Division von Potenzen mit gleicher Basis

$$\frac{10^4}{10^2} = 10^2 = 10^{4-2}$$

$$\frac{10^2}{10^4} = \frac{1}{10^2} = \frac{1}{10^{4-2}}$$

$$\frac{10^4}{10^4} = 1 = 10^{4-4} = 10^0 = 1$$

$$\frac{a^m}{a^p} = \begin{cases} a^{m-p}, & \text{wenn } m > p \\ 1, & \text{wenn } m = p \\ \frac{1}{a^{p-m}}, & \text{wenn } m < p \end{cases}$$

$$\text{Bsp.: } \frac{3^n}{3^{n-1}} = 3^{n-n+1} = 3$$

$$\frac{a^{2n-3}}{a^{2n+1}} = \frac{1}{a^{2n+1-2n+3}} = \frac{1}{a^4}$$

$$\frac{c^{2n-1}}{c^3} = c^{2n-1-3} = c^{2n-4}$$

Potenz einer Potenz

$$(3^4)^6 = 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 = 3^{4+4+4+4+4+4} = 3^{24} = 3^{4 \cdot 6}$$

$$\underline{(a^m)^p} = \underbrace{a^m \cdot a^m \cdot \dots \cdot a^m}_{p \text{ Faktoren}} = a^{\underbrace{m+m+\dots+m}_{p \text{ Faktoren}}} = a^{pm}$$

$$\text{Bsp.: } (3^7)^4 = 3^{28}$$

$$(b^3)^c = b^{3c}$$

$$(4^n)^{n+1} = 4^{n^2+n}$$

$$(c^{n+1})^{n-1} = c^{n^2-1}$$

$$(-4^{2n+1})^{2n} = 4^{4n^2+2n}$$

$$((-3)^{2n-1})^n = (-3)^{2n^2-n}$$

$$((-3)^{2n-2})^n = 3^{2n^2-2n}$$

$$7/11 \text{ a) } 5^2 + 3^4 = 25 + 81 = \underline{106} \checkmark$$

$$\text{b) } 4^3 + (-5)^4 = 64 + 625 = \underline{689} \checkmark$$

$$\text{c) } 6^3 - (-9)^3 = 216 + 729 = \underline{945} \checkmark \leftarrow$$

$$\text{d) } (-2)^8 - 3^5 = 2^8 - 3^5 = 256 - 243 = \underline{13} \checkmark$$

$$\text{e) } (-3)^5 - (-2)^7 = -3^5 + 2^7 = -243 + 128 = \underline{-115} \checkmark$$

$$\text{f) } (-10)^6 + (-10)^3 = 10^6 - 10^3 = 10^3(10^3 - 1) = \underline{10^3 \cdot 999} \checkmark$$

$$\text{g) } 10^{18} - (-10)^{16} = 10^{18} - 10^{16} = 10^{16}(10^2 - 1) = \underline{10^{16} \cdot 99} \checkmark$$

$$\text{h) } 10^5 + (-10)^9 = 10^5 - 10^9 = 10^5(1 - 10^4) = \underline{10^5 \cdot -9999} \checkmark \leftarrow$$

$$\text{i) } 10^4 + 10^5 + 10^6 - (-10)^7 = 10^4 + 10^5 + 10^6 + 10^7 = 10^4(1 + 10^1 + 10^2 + 10^3) = \underline{10^4 \cdot 1111} \checkmark$$

$$7/12 \text{ a) } (-1)^3 + (-2)^3 + (-3)^3 + (-4)^3 + (-5)^3 = \underline{-225} \checkmark$$

$$\text{b) } 9^3 + 7^3 + (-5)^4 + (-3)^5 - (-2)^6 = \underline{1390} \checkmark$$

$$\text{c) } 3^5 + (-4)^5 + 5^4 - (-6)^3 + (-7)^4 = \underline{2461} \checkmark$$

$$7/13 a) 2.5 \cdot 2^3 \cdot 4 \cdot 10^2 = \underline{8000} \checkmark \leftarrow$$

$$b) 0.5 \cdot 3^4 \cdot 20 \cdot 4^3 = \underline{51840} \checkmark$$

$$c) 0.1 \cdot 2^3 \cdot 1.2 \cdot 3^2 \cdot 10^4 = \underline{86400} \checkmark$$

$$7/14 a) \frac{6^3 + 3^2}{5^3 - 5^2} = \frac{225}{100} = \frac{9}{4} = \underline{2 \frac{1}{4}}$$

$$b) \frac{3^5 + 2^5}{4^3 + 6^2} = \frac{243 + 32}{64 + 36} = \frac{275}{100} = \frac{3}{2} = \cancel{\frac{1}{2}} = \underline{2 \frac{3}{4}}$$

$$c) \frac{2^8 - 3^4}{4^3 - 7^2} = \frac{256 - 81}{64 - 49} = \frac{175}{15} = \underline{11 \frac{2}{3}}$$

$$d) \frac{4^3 - 3^3}{5^3 - 4^3} = \frac{64 - 27}{125 - 64} = \frac{37}{61}$$

$$7/15 a) 9.6m^3 - 0.125m^3 - 3.25m^3 + 2.8n^3 = \underline{6.35m^3 + 2.675n^3}$$

$$7/16) \frac{2}{3}a^4 - \frac{3}{4}a^6 - \frac{4}{7}a^4 + \frac{5}{8}a^6 - \frac{1}{2}b^4 =$$

$$\frac{2}{3}a^4 - \frac{4}{7}a^4 + \frac{5}{8}a^6 - \frac{3}{4}a^6 - \frac{1}{2}b^4 =$$

$$\frac{14}{21}a^4 - \frac{12}{21}a^4 + \frac{5}{8}a^6 - \frac{6}{8}a^6 - \frac{1}{2}b^4 = \underline{\underline{\frac{2}{21}a^4 - \frac{1}{8}a^6 - \frac{1}{2}b^4}}$$

$$7/17) 7a^m + 8b^n - 5a^m - 20b^n + b^m - a^m + b^n - 3b^m =$$

$$7a^m - a^m - 5a^m + 8b^n - 20b^n + b^n + b^m - 3b^m = \underline{\underline{a^m - 11b^n - 2b^m}}$$

$$7/18) \frac{3}{4}x^m + \frac{2}{3}y^n + \frac{1}{2}x^m - \frac{5}{6}y^n + \frac{5}{8}x^m + \frac{1}{12}y^n =$$

$$\frac{3}{4}x^m + \frac{1}{2}x^m + \frac{5}{8}x^m + \frac{2}{3}y^n - \frac{5}{6}y^n + \frac{1}{12}y^n =$$

$$\frac{3}{4}x^m + \frac{2}{4}x^m + \frac{5}{8}x^m + \frac{8}{24}y^n - \frac{10}{24}y^n + \frac{1}{12}y^n =$$

$$\frac{6}{8}x^m + \frac{4}{8}x^m + \frac{5}{8}x^m + \frac{2}{3}y^n - \frac{10}{12}y^n + \frac{1}{12}y^n =$$

$$\frac{15}{8}x^m + \frac{2}{3}y^n - \frac{9}{12}y^n = \underline{\underline{\frac{7}{8}x^m + \frac{2}{3}y^n - \frac{3}{4}y^n}} = \frac{7}{8}x^m - \frac{1}{4}y^n$$

$$7/19) \frac{a}{b}x^m + \frac{c}{d}y^n - \frac{a}{2b}x^m - \frac{3c}{2d}y^n + \frac{3a}{4b}x^m - \frac{c}{3d}y^n + \frac{2a}{b}x^n - \frac{a}{3b}x^n =$$

$$\frac{a}{b}x^m - \frac{a}{2b}x^m + \frac{3a}{4b}x^m + \frac{c}{d}y^n - \frac{3c}{2d}y^n - \frac{c}{3d}y^n + \frac{2a}{b}x^n - \frac{a}{3b}x^n =$$

$$\frac{4a}{4b}x^m - \frac{2a}{4b}x^m + \frac{3a}{4b}x^m + \frac{6c}{6d}y^n - \frac{9c}{6d}y^n - \frac{2c}{6d}y^n + \frac{6a}{3b}x^n - \frac{a}{3b}x^n =$$

$$\frac{5a}{4b}x^m - \frac{5c}{6d}y^n - \frac{5a}{3b}x^n = \underline{\underline{5 \left(\frac{a}{4b}x^m - \frac{c}{6d}y^n - \frac{a}{3b}x^n \right)}}$$

7/20)

$$8(a+b)^2 - 6(a+b)^3 - 3(a-b)^2 + 9(a+b)^3 - (a+b)^2 + 7(a-b)^2 + (a+b)^3 =$$

$$8(a^2+2ab+b^2) - 6(a^3+a^2b+2a^2b+2ab^2+ab^2+b^3) - 3(a^2-2ab+b^2) + 9(a^3+a^2b+2a^2b+2ab^2+ab^2+b^3) - (a^2+2ab+b^2) + 7(a^2-2ab+b^2) + (a^3+a^2b+2a^2b+2ab^2+ab^2+b^3) =$$

$$\underline{\underline{7(a+b)^2 + 4(a+b)^3 + 4(a-b)^2}}$$

7/21)

$$c(a+b)^n + d(a+b)^n + e(a-b)^9 - (a+b)^n - f(a-b)^9 =$$

$$\underline{\underline{(a+b)^n \cdot (c+d-1) + (a-b)^9 \cdot (e-f)}}$$

$$7/22) \frac{a}{e}(x+y)^m - \frac{c}{d}(x+y)^m + \frac{e}{f}(x-y)^m - \frac{g}{h}(x-y)^m =$$

$$\underline{\underline{(x+y)^m \cdot \left(\frac{a}{e} - \frac{c}{d}\right) + (x-y)^m \cdot \left(\frac{e}{f} - \frac{g}{h}\right)}}$$

$$7/23) \frac{2a^n}{b^m} + \frac{3a^n}{2b^m} + \frac{4a^{n+1}}{3c^n} + \frac{6a^{n+1}}{c^n} - \frac{5a^{n+1}}{2c^n} =$$

$$\frac{4a^n}{2b^m} + \frac{3a^n}{2b^m} + \frac{8a^{n+1}}{6c^n} + \frac{36a^{n+1}}{6c^n} - \frac{15a^{n+1}}{6c^n} = \underline{\underline{\frac{7a^n}{2b^m} + \frac{29a^{n+1}}{6c^n}}}$$

$$7/24) \frac{5(a+b)^{n+1}}{(a-b)^n} - \frac{2(a+b)^{n+1}}{3(a-b)^n} + \frac{5(a+b)^n}{2(a-b)^m} - \frac{3(a+b)^n}{4(a-b)^m} =$$

$$\frac{15(a+b)^{n+1}}{3(a-b)^n} - \frac{2(a+b)^{n+1}}{3(a-b)^n} + \frac{10(a+b)^n}{4(a-b)^m} - \frac{3(a+b)^n}{4(a-b)^m} = \underline{\underline{\frac{13(a+b)^{n+1}}{3(a-b)^n} + \frac{7(a+b)^n}{4(a-b)^m}}}$$

$$8/33a) [(x+z)(x-z)]^2 = (x^2-z^2)^2 = (x^2-z^2)(x^2-z^2) = \underline{\underline{x^4 - 2x^2z^2 + z^4}} \quad \checkmark$$

$$b) [(x+z)(z-x)]^2 = (xz-x^2+z^2-zx)^2 = (z^2-x^2)^2 = (z^2-x^2)(z^2-x^2) = \underline{\underline{z^4 - 2xz^2 + x^4}} \quad \checkmark$$

$$c) [(x+z)(z-x)]^3 = (xz-x^2+z^2-zx)^3 = (z^2-x^2)^3 = (z^2-x^2)(z^2-x^2)^2 = (z^4 - 2z^2x^2 - z^2x^2 + x^4)(z^2-x^2) =$$

$$= (z^4 - 2z^2x^2 + x^4)(z^2-x^2) = z^6 - 2z^4x^2 + x^4z^2 - x^2z^4 + 2z^2x^4 - x^6 =$$

$$\underline{\underline{z^6 - 3x^2z^4 + 3x^4z^2 - x^6}} \quad \checkmark$$

$$8/34a) (b-a)^2 \equiv (a-b)^2$$

$$((-1)(a-b))^2 = (-1)^2(a-b)^2 = (a-b)^2$$

$$b) (b-a)^3 \equiv -(a-b)^3$$

$$((-1)(a-b))^3 = (-1)^3(a-b)^3 = -(a-b)^3$$

$$c) (b-a)^{2n} \equiv (a-b)^{2n}$$

$$((-1)(a-b))^{2n} = (-1)^{2n}(a-b)^{2n} = (a-b)^{2n}$$

$$d) (b-a)^{2n+1} \equiv -(a-b)^{2n+1}$$

$$((-1)(a-b))^{2n+1} = (-1)^{2n+1}(a-b)^{2n+1} = -(a-b)^{2n+1}$$

$$8/37) 9 \cdot \left(\frac{2}{3}\right)^3 + 8 \left(\frac{3}{4}\right)^2 + 27 \cdot \left(-\frac{1}{3}\right)^3 - 5 \left(\frac{3}{5}\right)^2 - 18 \cdot \left(\frac{5}{6}\right)^3 + 1000 \cdot \left(\frac{2}{5}\right)^3 =$$

$$9 \cdot \frac{8}{27} + 8 \cdot \frac{9}{16} + \left(27 \cdot \frac{1}{27}\right) - 5 \cdot \frac{9}{25} - 18 \cdot \frac{125}{216} + 1000 \cdot \frac{8}{125} =$$

$$2 \frac{2}{3} + 4 \frac{1}{2} - 31 - 1 \frac{4}{5} - 10 \frac{5}{12} + 64 =$$

$$\frac{2 \cdot 40}{60} + \frac{4 \cdot 30}{60} - 1 - \frac{1 \cdot 48}{60} - 10 \frac{25}{60} + 64 = 60$$

$$\frac{160 + 270 - 60 - 108 - 625 + 3840}{60} = \underline{\underline{57 \frac{19}{60}}}$$

$$8/38) 4 \cdot \left(\frac{1}{2}\right)^4 - 9 \left(\frac{2}{3}\right)^3 - 16 \cdot \left(\frac{1}{4}\right)^4 + 25 \cdot \left(\frac{1}{5}\right)^3 - 36 \left(\frac{1}{6}\right)^2 - 25 \cdot \left(-\frac{1}{10}\right)^2 =$$

$$4 \cdot \frac{1}{16} - 9 \cdot \frac{8}{27} - 16 \cdot \frac{1}{256} + 25 \cdot \frac{1}{125} - 36 \cdot \frac{1}{36} - 25 \cdot \frac{1}{100} =$$

$$\frac{1}{4} - 2 \frac{2}{3} - \frac{1}{16} + \frac{1}{5} - 1 - \frac{1}{4} =$$

$$\frac{60 - 640 - 15 + 48 - 240 - 60}{240} = \underline{\underline{-3 \frac{127}{240}}}$$

$$8/39) 9 \cdot \left(\frac{2}{3}\right)^2 + 15 \cdot \left(\frac{2}{5}\right)^3 - 36 \cdot \left(-\frac{5}{6}\right)^3 + 64 \cdot \left(-\frac{3}{4}\right)^4 - 64 \cdot \left(\frac{1}{2}\right)^6 =$$

$$9 \cdot \frac{4}{9} + 15 \cdot \frac{8}{125} - 36 \cdot \frac{125}{216} + 64 \cdot \frac{81}{256} - 64 \cdot \frac{1}{64} =$$

$$4 + \frac{24}{25} + 20 \frac{5}{6} + 20 \frac{1}{4} - 1 =$$

$$\frac{1200 + 288 + 6250 + 6075 - 300}{300} = \underline{\underline{45 \frac{13}{300}}}$$

$$8/40 a) \left(\frac{3a}{5b}\right)^3 \cdot \left(\frac{4c}{3}\right)^2 = \frac{27a^3}{125b^3} \cdot \frac{16c^2}{9} = \frac{432a^3c^2}{1125b^3} = \frac{48a^3c^2}{125b^3}$$

$$b) \left(\frac{2x}{3y}\right)^5 \cdot \left(\frac{3z}{4}\right)^3 = \frac{32x^5}{243y^5} \cdot \frac{27z^3}{64} = \frac{864x^5z^3}{15552y^5} = \frac{864x^5z^3}{18y^5}$$

$$c) \left(\frac{5a}{4b}\right)^4 \cdot \left(\frac{2c}{3d}\right)^3 = \frac{625a^4}{256b^4} \cdot \frac{8c^3}{27d^3} = \frac{5000a^4c^3}{6912b^4d^3} = \frac{625a^4c^3}{864b^4d^3}$$

$$9/41 a) \left(\frac{2m}{3n}\right)^5 \cdot \left(\frac{3n}{2m}\right)^6 \cdot \left(\frac{m}{n}\right)^1 = \frac{32m^5}{243n^5} \cdot \frac{729n^6}{64m^6} \cdot \frac{m}{n} = \frac{23328m^6n^6}{15552m^6n^6} = \frac{1}{2}$$

$$b) \left(\frac{2ab}{3c}\right)^3 \cdot \left(\frac{3c}{2b}\right)^4 \cdot \left(\frac{bc}{2a}\right)^5 = \frac{8a^3b^3}{27c^3} \cdot \frac{81c^4}{16b^4} \cdot \frac{b^5c^5}{32a^5} = \frac{648a^3b^8c^9}{13824a^5b^4c^3} = \frac{648b^4c^6}{13824a^2} = \frac{3b^4c^6}{64a^2}$$

$$c) \left(\frac{z}{x}\right)^3 \cdot \left(\frac{x}{z}\right)^2 \cdot (xy)^1 =$$

$$\left(\frac{z}{xy}\right)^3 \cdot \left(\frac{xy}{z}\right)^2 \cdot (xy)^1 = \frac{8}{x^3y^3} \cdot \frac{x^2y^2}{4} \cdot (xy)^1 = \frac{8x^2y^2}{4x^3y^3} \cdot (xy) = \frac{8x^3y^3}{4x^3y^3} = 2$$

$$d) \left(\frac{a}{b}\right)^5 \cdot \left(\frac{b}{a}\right)^3 \cdot \left(\frac{c}{a}\right)^2 =$$

$$\left(\frac{a}{bc}\right)^5 \cdot \left(\frac{bc}{a}\right)^3 \cdot \left(\frac{bc}{a}\right)^2 = \frac{a^5}{b^5c^5} \cdot \frac{b^3c^3}{a^3} \cdot \frac{b^2c^2}{a^2} = \frac{a^5b^5c^5}{a^5b^5c^5} = 1$$

$$e) \left(\frac{a+b}{a-b}\right)^3 \cdot \left(\frac{a-b}{a+b}\right)^2 \cdot \frac{1}{a^2+2ab+b^2} = \frac{(a+b)(a+b)(a+b)}{(a-b)(a-b)(a-b)} \cdot \frac{(a-b)(a-b)}{(a+b)(a+b)} \cdot \frac{1}{(a+b)(a+b)}$$

$$\frac{(a+b)(a+b)(a+b)(a-b)(a-b)}{(a-b)(a-b)(a-b)(a+b)(a+b)} = \frac{1}{(a-b)^2}$$

$$f) \left(\frac{1}{x+y}\right)^2 \cdot \left(\frac{x-y}{1}\right)^2 = \frac{1}{(x+y)(x-y)} \cdot \left(\frac{(x+y)(x-y)}{1}\right)^2 = \frac{1}{(x+y)(x-y)(x+y)(x-y)} \cdot \frac{(x+y)(x-y)(x+y)(x-y)}{1}$$

$$\frac{1}{(x+y)(x-y)} \cdot \frac{(x+y)(x-y)}{1} = (x+y)(x-y)$$

$$\underline{x^2 - y^2}$$

$$9/44) a) 25^3 \cdot 36^3 = (25 \cdot 36)^3 = 900^3 \checkmark$$

$$b) 12.5^4 \cdot 8^4 = (12.5 \cdot 8)^4 = 100^4 = \underline{\underline{10000000}} \cdot 10^5$$

$$c) 0.375^3 \cdot 8^3 = (8 \cdot 0.375)^3 = 3^3 = 27 \checkmark$$

$$d) \left(16 \frac{2}{3}\right)^3 \cdot 6^3 = \left(\frac{50}{3} \cdot 6\right)^3 = 100^3 \checkmark$$

$$9/45) a) \left(-3 \frac{1}{13}\right)^3 \cdot \left(-3 \frac{1}{4}\right)^3 = \left(-\frac{30}{13} \cdot -\frac{13}{4}\right)^3 = \left(\frac{30}{4}\right)^3 = \underline{\underline{\left(7 \frac{1}{2}\right)^3}} \quad 1000$$

$$b) \left(-5 \frac{1}{7}\right)^3 \cdot \left(8 \frac{1}{6}\right)^3 = \left(-\frac{36}{7} \cdot \frac{49}{6}\right)^3 = \left(-\frac{6}{1} \cdot \frac{7}{1}\right)^3 = \underline{\underline{-42^3}} \checkmark$$

$$c) \left(7 \frac{1}{2}\right)^5 \cdot \left(1 \frac{1}{3}\right)^5 = \left(\frac{15}{2} \cdot \frac{4}{3}\right)^5 = \left(\frac{5}{1} \cdot \frac{2}{1}\right)^5 = \underline{\underline{10^5}} \checkmark$$

$$9/49) a) 64 \cdot (-2x)^6 y^6 z^6 = 64 \cdot (-64x^6 y^6 z^6) = -64^2 x^6 y^6 z^6 = \underline{\underline{4096 x^6 y^6 z^6}} = \underline{\underline{(4xyz)^6}} \checkmark$$

$$b) 200 \cdot (-3a)^2 \cdot (-b)^2 \cdot (-c)^2 =$$

$$200 \cdot 9a^2 \cdot b^2 \cdot c^2 = \underline{\underline{1800 a^2 b^2 c^2}} = \underline{\underline{200 \cdot (3abc)^2}} \checkmark$$

$$c) -3000 \cdot (-m^3)^3 \cdot (-2n)^3 =$$

$$-3000 \cdot -m^3 \cdot -8n^3 = \underline{\underline{-24000 m^3 n^3}} = \underline{\underline{-3000 \cdot (2mn)^3}} \checkmark$$

$$d) (-a)^n \cdot (-a)^n = \underline{\underline{a^{2n}}} \checkmark$$

$$9/50c) \left(\frac{5}{13}\right)^3 \cdot \left(\frac{9}{17}\right)^3 = \left(\frac{68}{13} \cdot \frac{26}{17}\right)^3 = \left(\frac{4}{1} \cdot \frac{2}{1}\right)^3 = 8^3 \checkmark$$

$$9/51a) \left(\frac{2}{3}\right)^3 \cdot \left(-\frac{3}{2}\right)^3 = \left(\frac{2}{3} \cdot -\frac{3}{2}\right)^3 = \underline{\underline{-1}} \checkmark$$

$$b) \left(4\frac{5}{7}\right)^2 \cdot \left(1\frac{3}{11}\right)^2 = \left(\frac{33}{7} \cdot \frac{14}{11}\right)^2 = \left(\frac{3}{1} \cdot \frac{2}{1}\right)^2 = 6^2 = \underline{\underline{36}} \checkmark$$

$$c) \left(\frac{5}{9}\right)^8 \cdot \left(1\frac{4}{5}\right)^8 = \left(\frac{5}{9} \cdot \frac{9}{5}\right)^8 = \underline{\underline{1}} \checkmark$$

$$d) \left(1\frac{6}{7}\right)^9 \cdot \left(5\frac{5}{13}\right)^9 = \left(\frac{13}{7} \cdot \frac{70}{13}\right)^9 = \underline{\underline{10^9}} \checkmark$$

$$e) 2^7 \cdot \left(-\frac{1}{2}\right)^7 \cdot \left(-\frac{2}{3}\right)^7 \cdot \left(-\frac{3}{4}\right)^7 = \left(2 \cdot -\frac{1}{2} \cdot -\frac{2}{3} \cdot -\frac{3}{4}\right)^7 = \underline{\underline{\left(-\frac{1}{2}\right)^7}} \checkmark$$

$$9/53a) \left(\frac{2x}{3y}\right)^4 \cdot \left(\frac{9z}{8x}\right)^4 = \left(\frac{2x}{3y} \cdot \frac{9z}{8x}\right)^4 = \left(\frac{1}{y} \cdot \frac{3z}{4}\right)^4 = \underline{\underline{\left(\frac{3z}{4y}\right)^4}} \checkmark$$

$$b) \left(\frac{3x}{4y}\right)^5 \cdot \left(\frac{2y}{9}\right)^5 = \left(\frac{3x}{4y} \cdot \frac{2y}{9}\right)^5 = \left(\frac{2x}{2} \cdot \frac{1}{3}\right)^5 = \underline{\underline{\left(\frac{x}{6}\right)^5}} \checkmark$$

$$c) \left(\frac{2mn}{3pq}\right)^4 \cdot \left(\frac{9p}{8n}\right)^4 = \left(\frac{2mn}{3pq} \cdot \frac{9p}{8n}\right)^4 = \left(\frac{m}{q} \cdot \frac{3}{4}\right)^4 = \underline{\underline{\left(\frac{3m}{4q}\right)^4}} \checkmark$$

$$10/54a) \frac{3ab}{4cd} \cdot \left(-\frac{16d}{9a}\right)^3 = -\left(\frac{3ab}{4cd} \cdot \frac{16d}{9a}\right)^3 = \left(\frac{b}{c} \cdot -\frac{4}{3}\right)^3 = \underline{\underline{\left(\frac{4b}{3c}\right)^3}} \checkmark$$

$$b) -\left(\frac{2xy}{3z}\right)^4 \cdot \left(-\frac{9z}{8y}\right)^4 = -\left(\frac{2xy}{3z} \cdot \frac{9z}{8y}\right)^4 = -\left(\frac{x}{1} \cdot -\frac{3}{4}\right)^4 = \underline{\underline{\left(-\frac{3x}{4}\right)^4}} \checkmark$$

$$10/55a) \left(\frac{ax}{by}\right)^n \cdot \left(\frac{y}{x}\right)^n = \left(\frac{ax}{by} \cdot \frac{y}{x}\right)^n = \underline{\underline{\left(\frac{a}{b}\right)^n}} \checkmark$$

$$b) \left(-\frac{ab}{c}\right)^{2n} \cdot \left(\frac{cd}{a}\right)^{2n} = \left(\frac{ab}{c} \cdot \frac{cd}{a}\right)^{2n} = \left(\frac{b}{1} \cdot \frac{d}{1}\right)^{2n} = \underline{\underline{(bd)^{2n}}} \checkmark$$

$$c) -\left(-\frac{xyz}{mn}\right)^{2n+1} \cdot \left(\frac{m}{yz}\right)^{2n+1} = -\left(-\frac{xyz \cdot m}{mn \cdot yz}\right)^{2n+1} = \underline{\underline{\left(\frac{x}{n}\right)^{2n+1}}} \checkmark$$

$$10/55d) \left(\frac{a}{\frac{b}{c}}\right)^n \cdot \left(\frac{\frac{b}{c}}{a}\right)^n = \left(\frac{ac}{b} \cdot \frac{b}{ac}\right) = \underline{\underline{1}} \checkmark$$

$$e) \left(\frac{x^2-y^2}{xy}\right)^3 \cdot \left(\frac{xy}{\frac{x+y}{2}}\right)^3 = \left(\frac{(x+y)(x-y)}{xy} \cdot \frac{2xy}{x+y}\right)^3 = \frac{(2(x-y))^3}{1} = \underline{\underline{8(x-y)^3}} \checkmark$$

$$f) \left(\frac{a-b}{c}\right)^n \cdot \left(\frac{abc}{a^2-b^2}\right)^n \cdot \left(1+\frac{a}{b}\right)^n = \left(\frac{a-b}{c} \cdot \frac{abc}{(a+b)(a-b)} \cdot \frac{a+b}{b}\right)^n = \underline{\underline{a^n}} \checkmark$$

$$10/63a) \left(\frac{2}{3}\right)^3 : \left(\frac{4}{9}\right)^3 = \left(\frac{2}{3} \cdot \frac{9}{4}\right)^3 = \left(\frac{3}{1} \cdot \frac{3}{4}\right)^3 = \left(\frac{9}{4}\right)^3 = \frac{2}{3} \cdot \frac{9}{4} = \frac{1}{1} \cdot \frac{3}{2} = \underline{\underline{\left(\frac{1}{2}\right)^3}} \checkmark$$

$$b) \left(\frac{3}{4}\right)^4 \cdot \left(\frac{9}{8}\right)^4 = \left(\frac{3}{4} \cdot \frac{9}{8}\right)^4 = \left(\frac{1}{1} \cdot \frac{2}{3}\right)^4 = \underline{\underline{\left(\frac{2}{3}\right)^4}} \checkmark$$

$$c) \left(\frac{3}{7}\right)^2 : \left(1\frac{4}{35}\right)^2 = \left(\frac{3}{7} \cdot \frac{39}{4}\right)^2 = \left(\frac{3}{7} \cdot \frac{4}{39}\right)^2 = \left(\frac{1}{7} \cdot \frac{4}{13}\right)^2 = \underline{\underline{\left(\frac{4}{91}\right)^2}} \checkmark$$

$$64a) \left(\frac{1}{3}\right)^6 : \left(-\frac{3}{2}\right)^6 = \left(\frac{1}{3} \cdot -\frac{2}{3}\right)^6 = \underline{\underline{\left(\frac{2}{9}\right)^6}} \checkmark$$

$$b) \left(\frac{5}{3}\right)^2 : \left(1\frac{5}{6}\right)^2 = \left(\frac{5}{3}\right)^2 : \left(\frac{11}{6}\right)^2 = \left(\frac{5}{3} \cdot \frac{6}{11}\right)^2 = \left(\frac{55}{18}\right)^2 = \underline{\underline{\left(3\frac{1}{18}\right)^2}} \checkmark$$

$$c) \left(-\frac{2}{3}\right)^5 \cdot \left(-\frac{4}{3}\right)^5 = \left(-\frac{2}{3} \cdot -\frac{4}{3}\right)^5 = \underline{\underline{\left(\frac{1}{2}\right)^5}} \checkmark$$

$$65a) \left(\frac{a}{b}\right)^n \cdot \left(\frac{c}{d}\right)^n = \left(\frac{a}{b} \cdot \frac{d}{c}\right)^n = \underline{\underline{\left(\frac{ad}{bc}\right)^n}} \checkmark$$

$$b) \left(\frac{x}{y}\right)^{2n} : \left(-\frac{x}{y}\right)^{2n} = \left(\frac{x}{y} \cdot -\frac{y}{x}\right)^{2n} = \underline{\underline{+1}} \checkmark$$

$$c) \left(\frac{2p}{3q}\right)^{2n+1} : \left(-\frac{2p}{3q}\right)^{2n+1} = \left(\frac{2p}{3q} \cdot -\frac{3q}{2p}\right)^{2n+1} = \underline{\underline{-1}} \checkmark$$

$$66a) \left[\left(\frac{5}{6}\right)^4 : \left(\frac{3}{2}\right)^4\right] : \left(\frac{4}{9}\right)^4 = \left(\frac{5}{6} \cdot \frac{2}{3}\right)^4 : \left(\frac{4}{9}\right)^4 = \left(\frac{5}{9}\right)^4 : \left(\frac{4}{9}\right)^4 = \left(\frac{5 \cdot 9}{9 \cdot 4}\right)^4 = \left(\frac{5}{4}\right)^4 = \underline{\underline{\left(1\frac{1}{4}\right)^4}} \checkmark$$

$$b) \left[3\frac{13}{14}\right]^5 : \left(2\frac{5}{14}\right)^5 = \left[\left(\frac{55}{14} \cdot \frac{14}{33}\right)\right]^5 : \left(\frac{5}{6}\right)^5 = \left(1\frac{5}{3} \cdot \frac{6}{5}\right)^5 = 2^5 = \underline{\underline{32}} \checkmark$$

$$64b) \left(\frac{5}{3}\right)^2 : \left(1\frac{5}{6}\right)^2 = \left(\frac{5}{3} \cdot \frac{26}{11}\right)^2 = \underline{\underline{\left(\frac{10}{11}\right)^2}} \checkmark$$

$$11/67a) \left[\left(\frac{a+b}{a-b} \right)^m \cdot \left(\frac{a-b}{a+b} \right)^m \right] : \left(\frac{b}{a} \right)^m = \left(\frac{(a+b)(a-b)}{(a-b)(a+b)} \right)^m \cdot \left(\frac{b}{a} \right)^m = \left(1 \cdot \frac{a}{b} \right)^m = \underline{\underline{\left(\frac{a}{b} \right)^m}} \quad \checkmark$$

$$b) \left[\left(\frac{x+y}{a-b} \right)^n \cdot \left(\frac{a^2-b^2}{x^2-y^2} \right)^n \right] \cdot \left(\frac{a+b}{x-y} \right)^n = \left(\frac{(x+y)(a+b)(b-b)}{(a-b)(x+y)(x-y)} \right)^n \cdot \frac{x-y}{a+b} = \underline{\underline{1}} \quad \checkmark$$

$$c) \left[\left(\frac{a+b}{c+d} \right)^4 \cdot \left(\frac{a+b}{c-d} \right)^4 \right] : \left(\frac{c-d}{a+b} \right)^4 = \left(\frac{(a+b)(c-d)}{(a+b)(c+d)} \cdot \frac{a+b}{c-d} \right)^4 = \underline{\underline{\left(\frac{a+b}{c+d} \right)^4}} \quad \checkmark$$

$$d) \left[\left(\frac{x}{x+y} \right)^n \cdot \left(\frac{x^2-y^2}{x+y} \right)^n \right] : \left(\frac{1}{x+y} \right)^n = \left(\frac{x(x+y)}{(x+y)(x+y)(x-y)} \cdot \frac{x+y}{1} \right)^n = \underline{\underline{\left(\frac{x}{x-y} \right)^n}} \quad \checkmark$$

$$11/68a) (2a \pm b)^2 = \begin{cases} (2a+b)(2a+b) = 4a^2 + 4ab + b^2 \\ (2a-b)(2a-b) = 4a^2 - 4ab + b^2 \end{cases} \quad \checkmark$$

$$b) (3x \pm 2y)^2 = \begin{cases} (3x+2y)(3x+2y) = 9x^2 + 12xy + 4y^2 \\ (3x-2y)(3x-2y) = 9x^2 - 12xy + 4y^2 \end{cases} \quad \checkmark$$

$$c) \left(\frac{3}{a} \pm \frac{4}{b} \right)^2 = \begin{cases} \left(\frac{3}{a} + \frac{4}{b} \right) \left(\frac{3}{a} + \frac{4}{b} \right) = \frac{9}{a^2} + \frac{24}{ab} + \frac{16}{b^2} \\ \left(\frac{3}{a} - \frac{4}{b} \right) \left(\frac{3}{a} - \frac{4}{b} \right) = \frac{9}{a^2} - \frac{24}{ab} + \frac{16}{b^2} \end{cases} \quad \checkmark$$

$$11/69a) \left(\frac{a}{b} \pm \frac{b}{a} \right)^2 = \left(\frac{a}{b} + \frac{b}{a} \right) \left(\frac{a}{b} + \frac{b}{a} \right) = \underline{\underline{\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}}}$$

$$\frac{a}{b} - \left(\frac{a}{b} - \frac{b}{a} \right) \left(\frac{a}{b} - \frac{b}{a} \right) = \underline{\underline{\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}}} \quad \checkmark$$

$$b) \left(\frac{x}{2y} + \frac{y}{2x} \right)^2 = \left(\frac{1}{2} + \frac{1}{2} \right) \left(\frac{1}{2} + \frac{1}{2} \right) = \underline{\underline{\left(\frac{x}{2y} + \frac{y}{2x} \right)^2 = \frac{x^2}{4y^2} + \frac{1}{2} + \frac{y^2}{4x^2}}} \quad \checkmark$$

$$c) \left(\frac{2m}{3} + \frac{3n}{2} \right)^2 = \left(\frac{2m}{3} + \frac{3n}{2} \right)^2 = (mn)(mn) = \underline{\underline{\frac{4m^2}{9} + 2mn + \frac{9n^2}{4}}}$$

$$11/70a) (2a \pm b)^3 = (2a+b)(2a+b)(2a+b) = (4a^2 + 4ab + b^2)(2a+b) = 8a^3 + 8a^2b + 2ab^2 + 4a^2b + 4ab^2 + b^3 = \underline{\underline{8a^3 + 12a^2b + 6ab^2 + b^3}} \quad \checkmark$$

$$(2a-b)(2a-b)(2a-b) = (4a^2 - 4ab + b^2)(2a-b) = 8a^3 - 8a^2b + 2ab^2 - 4a^2b + 4ab^2 - b^3 = \underline{\underline{8a^3 - 12a^2b + 6ab^2 - b^3}} \quad \checkmark$$

$$69c) \left(\frac{2m}{3} + \frac{3n}{2} \right)^2 = \frac{4m^2}{9} + 2 \frac{6mn}{6} + \frac{9n^2}{4} = \underline{\underline{\frac{4m^2}{9} + 2mn + \frac{9n^2}{4}}}$$

$$11/70b) (3x+2y)^3 = (3x+2y)(3x+2y)(3x+2y) = (9x^2+12xy+4y^2)(3x+2y) =$$

$$27x^3+36x^2y+12xy^2+18x^2y+24xy^2+8y^3 =$$

$$\underline{\underline{27x^3+52x^2y+36xy^2+8y^3}}$$

$$= (3x-2y)(3x-2y)(3x-2y) = (9x^2-12xy+4y^2)(3x-2y) =$$

$$\underline{\underline{27x^3-52x^2y+36xy^2-8y^3}}$$

$$c) \left(\frac{1}{2}x + \frac{2}{3}y\right)^3 = \left(\frac{1}{2}x + \frac{2}{3}y\right)\left(\frac{1}{2}x + \frac{2}{3}y\right)\left(\frac{1}{2}x + \frac{2}{3}y\right) =$$

$$\left(\frac{1}{4}x^2 + \frac{2}{6}xy + \frac{2}{6}xy + \frac{4}{9}y^2\right)\left(\frac{1}{2}x + \frac{2}{3}y\right) =$$

$$\left(\frac{1}{8}x^3 + \frac{2}{12}x^2y + 2\frac{2}{12}xy^2 + 2\frac{4}{18}xy^2 + \frac{4}{18}xy^2 + \frac{8}{27}y^3\right) =$$

$$\left(\frac{1}{8}x^3 + \frac{2}{12}x^2y + \frac{4}{18}xy^2 + \frac{8}{27}y^3\right) = \underline{\underline{\frac{1}{8}x^3 + \frac{2}{3}x^2y + \frac{4}{9}xy^2 + \frac{8}{27}y^3}}$$

Verbesserung

$$10/63c) \left(\frac{3}{7}\right)^2 : \left(\frac{4}{35}\right)^2 = \left(\frac{3}{7} \cdot \frac{35}{4}\right)^2 = \left(\frac{1}{1} \cdot \frac{5}{1}\right)^2 = \underline{\underline{\left(\frac{5}{1}\right)^2}}$$

$$a) \left(\frac{2}{3}\right)^3 : \left(\frac{4}{9}\right)^3 = \left(\frac{2}{3} \cdot \frac{9}{4}\right)^3 = \left(\frac{1}{1} \cdot \frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^3 = \underline{\underline{\left(\frac{1}{2}\right)^3}}$$

$$10/64b) \left(\frac{5}{3}\right)^2 : \left(\frac{6}{11}\right)^2 = \left(\frac{5}{3} \cdot \frac{11}{6}\right)^2 = \left(\frac{5}{1} \cdot \frac{11}{2}\right)^2 = \underline{\underline{\left(\frac{10}{1}\right)^2}}$$

$$11/69b) \left(\frac{x}{2y} + \frac{y}{2x}\right)^2 = \frac{x^2}{4y^2} + \frac{y^2}{4x^2} + 2\frac{xy}{4xy} = \underline{\underline{\frac{x^2}{4y^2} + \frac{1}{2} + \frac{y^2}{4x^2}}}$$

$$c) \left(\frac{2m}{3} + \frac{3n}{2}\right)^2 = \frac{4m^2}{9} + \frac{9n^2}{4} + 2\frac{6mn}{6} = \underline{\underline{\frac{4m^2}{9} + \frac{9n^2}{4} + 2mn}}$$

$$11/71a) \left(\frac{a}{b} + \frac{b}{a}\right)^3 = \frac{a^3}{b^3} + \frac{3a^2b}{ab^2} + \frac{3ab^2}{a^2b} + \frac{b^3}{a^3}$$

$$\left(\frac{a}{b} + \frac{b}{a}\right)\left(\frac{a}{b} + \frac{b}{a}\right) = \left(\frac{a^2}{b^2} + 2\frac{ab}{ab} + \frac{b^2}{a^2}\right)$$

$$\left(\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}\right)\left(\frac{a}{b} + \frac{b}{a}\right) = \frac{a^3}{b^3} + \frac{a^2b}{ab^2} + 2\frac{a}{b} + 2\frac{b}{a} + \frac{ab^2}{a^2b} + \frac{b^3}{a^3} =$$

$$\underline{\underline{\frac{a^3}{b^3} + 3\frac{a}{b} + 3\frac{b}{a} + \frac{b^3}{a^3}}}$$

$$b) \left(\frac{x}{2y} + \frac{y}{x}\right)^3 = \left(\frac{a}{b} + \frac{b}{a}\right)\left(\frac{a}{b} + \frac{b}{a}\right) = \frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$$

$$\left(\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}\right)\left(\frac{a}{b} - \frac{b}{a}\right) = \frac{a^3}{b^3} - \frac{a^2b}{ab^2} - 2\frac{a}{b} + 2\frac{b}{a} + \frac{ab^2}{a^2b} - \frac{b^3}{a^3} =$$

$$\underline{\underline{\frac{a^3}{b^3} - 3\frac{a}{b} + 3\frac{b}{a} - \frac{b^3}{a^3}}}$$

$$b) \left(\frac{x}{2y} + \frac{2y}{x}\right)^3 = \left(\frac{x^2}{4y^2} + \frac{4y^2}{x^2} + 2\frac{2xy}{2xy}\right)\left(\frac{x}{2y} + \frac{2y}{x}\right) = \frac{x^3}{8y^3} + \frac{8y^3}{x^3} + 3\frac{x}{2y} + 3\frac{2y}{x}$$

$$c) \left(\frac{3m}{4} + \frac{4m}{3}\right)^3 = \frac{27m^3}{64} + 3\frac{3m}{4} + 3\frac{4m}{3} + \frac{64m^3}{27} \quad (\checkmark)$$

$$11/72a) (a+b+3c)^2 = (a+b+3c)(a+b+3c) = a^2 + ab + 3ac + ab + b^2 + 3bc + 3ac + 3bc + 9c^2 = \underline{\underline{a^2 + 2ab + 6ac + 6bc + b^2 + 9c^2}}$$

$$b) (2a-3b+4c)^2 = (2a-3b+4c)(2a-3b+4c) = 4a^2 - 6ab + 8ac - 6ab + 9b^2 - 12bc + 8ac - 12bc + 16c^2 = \underline{\underline{4a^2 - 12ab + 16ac - 24bc + 9b^2 + 16c^2}}$$

$$c) (1+x+y)^2 = (1+x+y)(1+x+y) = 1 + x + y + x + x^2 + xy + y + xy + y^2 = \underline{\underline{1 + x^2 + 2x + 2y + 2xy + y^2}}$$

$$(1+x-y)^2 = (1+x-y)(1+x-y) = 1 + x - y + x + x^2 - xy - y - xy + y^2 = \underline{\underline{1 + x^2 + 2x - 2y - 2xy + y^2}}$$

$$11/73 a) (a^2 - b^2) : (a \pm b)^2 = (a+b)(a-b) : (a \pm b)(a \pm b) =$$

$$\frac{(a+b)(a-b)}{(a \pm b)(a \pm b)} \quad \text{od.} \quad \frac{(a+b)(a-b)}{(a-b)(a-b)} = \underline{\underline{(a \mp b) : (a \pm b)}}$$

$$b) (9x^2 - 4y^2)^2 : (3x \pm 2y)^2 = (3x+2y)(3x-2y)(3x+2y)(3x-2y) : (3x \pm 2y)^2 =$$

$$\frac{(3x+2y)(3x+2y)(3x-2y)(3x-2y)}{(3x+2y)(3x+2y)} \quad \text{od.} \quad \frac{(3x+2y)(3x+2y)(3x-2y)(3x-2y)}{(3x-2y)(3x-2y)}$$

$$= \underline{\underline{(3x \pm 2y)^2}} \quad \checkmark$$

$$c) (a^2 - b^2)^3 : (a \pm b)^3 = [(a+b)(a-b)]^3 : (a \pm b)^3 =$$

$$\frac{(a+b)(a+b)(a+b)(a-b)(a-b)(a-b)}{(a \pm b)(a \pm b)(a \pm b)} \quad \text{od.} \quad \frac{(a+b)(a+b)(a+b)(a-b)(a-b)(a-b)}{(a-b)(a-b)(a-b)}$$

$$\underline{\underline{(a \pm b)^3}} \quad \checkmark$$

$$d) (4x^2 - 9y^2)^3 : (2x \pm 3y)^3 = [(2x+3y)(2x-3y)]^3 : (2x \pm 3y) =$$

$$\frac{(2x+3y)(2x+3y)(2x+3y)(2x-3y)(2x-3y)(2x-3y)}{(2x+3y)(2x+3y)(2x+3y)} \quad \text{od.} \quad \frac{(2x+3y)(2x+3y)(2x+3y)(2x-3y)(2x-3y)(2x-3y)}{(2x-3y)(2x-3y)(2x-3y)}$$

$$= \underline{\underline{(2x \pm 3y)^3}} \quad \checkmark$$

$$13/100) 2(a-b) \cdot \frac{1}{24}(a-b)^{n+2} \cdot 3(a-b)^{n-1} \cdot \frac{2}{3}(b-a)^{2n+1} =$$

$$(a-b) \cdot \frac{1}{24}(a-b)^{n+2} \cdot (a-b)^{n-1} \cdot 2(b-a)^{2n+1} =$$

$$(a-b) \cdot (a-b)^{n+2} \cdot (a-b)^{n-1} \cdot (b-a)^{2n+1} = (a-b)^{2n+1} \cdot (b-a)^{2n+1} =$$

$$\underline{\underline{-(a-b)^{4n+2}}} \quad \checkmark$$

$$13/101a) \frac{4a^4b^3}{c^5} \cdot \frac{a^2b^5c^5}{2} = \frac{4a^6b^8c^5}{2c^5} = \underline{\underline{2a^6b^8}} \quad \frac{2a^3b^4}{3} \cdot \frac{3a^2b}{4} = \frac{6a^5b^5}{12} = \underline{\underline{\frac{(ab)^5}{2}}} \quad \checkmark$$

$$b) \frac{3x^2y^5}{5} \cdot \frac{10x^3y^4}{9} = \frac{30x^5y^9}{45} = \underline{\underline{\frac{2x^5y^9}{3}}}$$

$$c) \frac{5m^2n^3}{6} \cdot \frac{3mn^2}{4} = \frac{15m^3n^5}{24} = \underline{\underline{\frac{5m^3n^5}{8}}} \quad \checkmark$$

$$13/102a) \frac{4a^4b^3}{c^5} \cdot \frac{a^2b^5c^5}{2} = \frac{4a^6b^8c^5}{2c^5} = \underline{\underline{2a^6b^8}} \quad \checkmark$$

$$13/102b) \frac{3m^{x+1} \cdot n^{y-1}}{2p} \cdot \frac{2m^{2x-1} \cdot n^{y+1}}{3p^2} = \frac{(6m)^{3x} \cdot (n^2)^y}{6p^3} = \frac{(6m^2n^2)^{3x+2y}}{6p^3}$$

$$13/103a) \frac{a^n}{(a+b)^2} \cdot \frac{a^{n+2}}{(a+b)^3} = \frac{(a^2)^{2n+2}}{(a+b)^5} \quad \checkmark$$

$$b) \frac{(a-b)^3 \cdot (a+b)^2}{(c+d)^2} \cdot \frac{(a+b)^3 \cdot (b-a)^3}{a(c+d)^3} = \frac{(a^3 - 2a^2b + 2ab^2 - b^3)(a+b)(a^2 + 2ab + b^2)}{a(c+d)^5}$$

$$\frac{(a^3 + 2a^2b + 2ab^2 + b^3) \cdot (b^3 - 2b^2a + 2ba^2 - a^3)}{a(c+d)^5} =$$

$$\frac{(a+b)^5 \cdot (b-a)^6}{a(c+d)^5}$$

$$13/104a) (x^2 - x^3) \cdot x^4 = \underline{x^6 - x^7} \quad \checkmark$$

$$b) (y + y^4 - y^7) \cdot y^5 = y^6 + y^9 - y^{12} \quad \checkmark$$

$$c) (2z^2 + 3z^3 - 4z^4) \cdot 5z^6 = 10z^8 + 15z^9 - 20z^{10} \quad \checkmark \quad \underline{5(2z^8 + 3z^9 - 4z^{10})}$$

$$13/105a) \left(\frac{1}{2}a^2 - \frac{1}{4}a^3 + \frac{3}{4}a^5\right) \cdot 4a^4 = \underline{2a^6 - a^7 + 3a^9} \quad \checkmark$$

$$b) \left(\frac{1}{3}x^3 + \frac{1}{4}x^4 - \frac{2}{3}x^5\right) \cdot 12x^2 = \underline{4x^5 + 3x^6 - 8x^7} \quad \checkmark$$

$$13/106a) (x^m - x^n) \cdot x^p = \underline{(x^2)^{m+p} - (x^2)^{n+p}} \quad \checkmark$$

$$b) (y^{m+1} + y^{m+2}) \cdot y^{n-1} = \underline{(y^2)^{n+m} + (y^2)^{m+n+1}} \quad \checkmark$$

$$c) (z^n - z^{2n-1}) \cdot z^{3n+2} = \underline{(z^2)^{4n+2} - (z^2)^{5n+1}} \quad \checkmark$$

$$13/107a) (a^2 + a^3) \cdot (1+a) = a^2 + a^3 + a^3 + a^4$$

$$b) (3b^3 + 4b^4) \cdot (b^3 - b^2) = 3b^6 - 3b^5 + 4b^7 - 4b^6 = 4b^7 - 3b^5 - b^6$$

$$c) (5c - 2c^3) \cdot (2c^2 + 3c^4) = 10c^3 + 15c^5 - 6c^7$$

$$13/108a) (a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6) \cdot (a-b) =$$

$$a^7 + a^6b + a^5b^2 + a^4b^3 + a^3b^4 + a^2b^5 + ab^6 - a^6b - a^5b^2 - a^4b^3 - a^3b^4 - a^2b^5 - ab^6 - b^7$$

$$= \underline{a^7 - b^7} \quad \checkmark$$

$$13/109 a) (a^4 + a^2b^2 + b^4) \cdot (a^2 - b^2) = \cancel{(a^2+b^2)(a^2-b^2)} \\ = a^6 + a^4b^2 + a^2b^4 - a^4b^2 - a^2b^6 - b^6 = \underline{\underline{a^6 - b^6}} \checkmark$$

$$b) (a^6 + a^3b^3 + b^6) \cdot (a^3 - b^3) = \\ a^9 + a^6b^3 + a^3b^6 - a^6b^3 - a^3b^6 - b^9 = \underline{\underline{a^9 - b^9}} \checkmark$$

$$13/110 (1+x^2+x^3) \cdot (1-x^2-x^3) = 1 - x^2 - x^3 + x^2 - x^4 - x^5 + x^3 - x^5 - x^6 = \\ \underline{\underline{1 - x^4 - 2x^5 - x^6}} \checkmark$$

$$13/111 (a^3 + 3a^2b + 3ab^2 + b^3) \cdot (a^3 - 3a^2b + 3ab^2 - b^3) = \\ (a+b)^3 \cdot (a-b)^3 = \underline{\underline{(a-b)^6}} \underline{\underline{[(a+b)(a-b)]^6}}$$

$$13/112 (16x^4 - 48x^3y + 108x^2y^2 - 108xy^3 + 81y^4) \cdot (4x^2 + 12xy + 9y^2) = \\ \underline{\underline{(4x^2 + 12xy + 9y^2)^3}} \quad (4x^6 + 102x^5y + 144x^4y^2)$$

$$13/113 (a^8 + a^7b + a^6b^2 + a^5b^3 + a^4b^4 + a^3b^5 + a^2b^6 + ab^7 + b^8) \cdot (a^4 - a^3b + ab^3 - b^4) = \\ \underline{\underline{(a^4 - a^3b + ab^3 - b^4)^3}}$$

$$13/114 a) (a^n + b^n)(a^n - b^n) = (a^{2n} - ab^{2n} + ab^{2n} - (b^{2n})^{2n}) = (a^{2n})^{2n} - (b^{2n})^{2n} = \underline{\underline{(a^2 - b^2)^{2n}}} \checkmark$$

$$b) [(a+b)^{m+1} + (a+b)^{m-1}] \cdot (a+b) = [(a+b)^{m+1}]^{m+1} + [(a+b)^{m-1}]^{m-1}$$

$$13/115 (x^{2mn} + x^m + x^n + x^{2n-m})(x^m - x^n) = (x^{2m-n} + (x^{2m})^{2n} + (x^{2n})^{2m} + (x^2)^{2n}) - \\ (x^2)^{2m} - (x^2)^{2mn} - (x^2)^{2n} - (x^2)^{3n-m} = \underline{\underline{(x^2)^{3m-n} - (x^2)^{3n-m}}} \checkmark$$

$$13/116 (a^{3m-n} - a^{2m} + a^{m+n} - a^{2n} + a^{3n-m}) \cdot (a^m + a^n) = \\ (a^2)^{3m-n} + a^{2m} + a^{2m+n} - a^{m+2n} + a^{3n} + a^{3n} - a^{2m+n} + a^{m+2n} + a^{3n} + a^{4n-m} = \\ \underline{\underline{a^{4m-n} + a^{4n-m}}}$$

$$13/117 (2a^m - 3a^n) \cdot (2a^m + 3a^n) \cdot (a^m + a^n) \cdot (a^m - a^n) = \\ (4a^{2m} - 9a^{2n})(a^{2m} - a^{2n}) = \underline{\underline{4a^{4m} + 9a^{2m+2n} + 4a^{2m+2n} + 9a^{4n}}} = \\ \underline{\underline{4a^{4m} + 13a^{2m+2n} + 9a^{4n}}}$$

$$13/118 \text{ a) } (x^2+1)^2 \cdot (3x^2-1) = \cancel{3x^4} \cancel{x^2} +$$

$$(x^4+2x^2+1) \cdot (3x^2-1) = 3x^6 - x^4 + 6x^4 - 2x^2 + 3x^2 + 1 =$$

$$\underline{\underline{3x^6 + 5x^4 + x^2 + 1}}$$

$$\text{b) } (a+b)^2 \cdot (a-b) = (a^2+2ab+b^2)(a-b) = a^3+2a^2b+ab^2-a^2b-2ab^2-b^3 \\ = \underline{\underline{a^3+a^2b-ab^2-b^3}}$$

$$\text{c) } (x+y)^3 \cdot (x-y) = (x^3+2x^2y+xy^2+x^2y+2xy^2+y^3) \cdot (x-y) \\ = x^4+3x^3y+3x^2y^2+xy^3-x^3y-3x^2y^2-3xy^3-y^4 \\ = \underline{\underline{x^4+2x^3y-2xy^3-y^4}}$$

$$13/119 \text{ a) } \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \cdot \left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \frac{1}{a^4 - b^4}$$

$$\text{b) } \left(\frac{a^2}{2} + \frac{ab}{9} - \frac{b^2}{4}\right) \cdot \left(\frac{a}{2} - \frac{b}{3}\right) = \frac{(18a^2+4ab-9b^2)(18a-12b)}{36} =$$

$$\frac{324a^3+72a^2b-162ab^2-216a^2b-48ab^2+108b^3}{36} =$$

$$\frac{324a^3-144a^2b-210ab^2+108b^3}{36} =$$

$$\frac{108a^3-48a^2b-70ab^2+36b^3}{12}$$

$$\begin{array}{l} 324 | 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ 144 | 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \\ 210 | 2 \cdot 3 \cdot 5 \cdot 7 \\ 108 | 3 \cdot \dots \end{array}$$

$$(a^2) \quad (b^2) \quad (b)$$

$$13/120 \text{ a) } (1.2x^a+1.5y^b) \cdot (1.2x^a-1.5y^b) =$$

$$\underline{\underline{(1.44x^{2a} - 2.25y^{2b})}}$$

$$\text{b) } (1.2a^4+0.3a^5-0.1a^6) \cdot (0.3a^3-0.2a^4) =$$

$$0.36a^7 - 0.06a^8 + 0.09a^8 - 0.06a^9 - 0.03a^9 + 0.02a^{10} =$$

$$\underline{\underline{0.36a^7 + 0.03a^8 - 0.09a^9 + 0.02a^{10}}}$$

$$12/93a) \frac{1}{2}(4a)^2 \cdot \frac{1}{4}(-3a)^3 \cdot \frac{2}{9}(-2a)^4 =$$

$$\frac{1}{36}(4a)^2 \cdot (-3a)^3 \cdot (-2a)^4 = \frac{1}{36} 16a^2 \cdot 27a^3 \cdot 16a^4 = \frac{6912a^9}{36}$$

$$= \underline{\underline{192a^9}}$$

$$b) \frac{3}{4}(xyz)^2 \cdot \frac{2}{3}(2xy)^3 \cdot \frac{1}{8}(-3x)^2 = \frac{6}{96} x^2 y^2 \cdot 8x^3 y^3 \cdot 9x^2 =$$

$$\frac{6 \cdot 72 x^7 y^5 z^2}{96} = \frac{432 x^7 y^5 z^2}{96} =$$

$$12/94a) 1.2a \cdot 0.04a^3 \cdot 0.6a^n = \underline{\underline{0.0288a^{4+n}}}$$

$$b) 2.5(ab)^2 \cdot 0.5(2ab)^4 \cdot 0.3a^3 =$$

$$2.5a^2 b^2 \cdot 1a^4 b^4 \cdot 0.3a^3 = 0.75a^9 b^6$$

$$12/95a) \frac{a}{b} x^{2n} \cdot (-x)^{2n+1} \cdot (-x)^{2n} =$$

$$\frac{a}{b} x^{2n} \cdot -x^{2n+1} \cdot x^{2n} = \underline{\underline{-\frac{a}{b} x^{6n+1}}}$$

$$b) \frac{c}{d} y^{3n-2} \cdot (-y)^{2n-1} \cdot (-y)^{2n+1} =$$

$$\frac{c}{d} y^{3n-2} \cdot -y^{2n-1} \cdot y^{2n+1} = \underline{\underline{\frac{c}{d} y^{7n-2}}}$$

$$12/96) \frac{2}{3} a^{m+1} \cdot \frac{1}{2} b^{n+2} \cdot \frac{3}{4} a^{m+2} \cdot \frac{1}{2} b^{n-1} \cdot 2a^{m-2} \cdot \frac{2}{8} b^{n+3} =$$

$$a^{m+1} \cdot b^{n+2} \cdot a^{m+2} \cdot b^{n-1} \cdot a^{m-2} \cdot 2b^{n+3} =$$

$$\underline{\underline{a^{3m+1} \cdot 2b^{3n+4}}}$$

$$13/100) 2(a-b) \cdot \frac{1}{4}(a-b)^{n+2} \cdot 3(a-b)^{n-1} \cdot \frac{2}{3}(b-a)^{2n+1} =$$

$$2(a-b) \cdot \frac{1}{4}(a-b)^{n+2} \cdot 3(a-b)^{n-1} \cdot -\frac{2}{3}(a-b)^{2n+1} =$$

$$-\frac{2}{2}(a-b) \cdot (a-b)^{n+2} \cdot (a-b)^{n-1} \cdot (a-b)^{2n+1} = \underline{\underline{-(a-b)^{4n+2}}}$$

Verbesserung

$$13/107a) (a^2+a^3)(1+a) = a^2+a^3+a^3+a^4 = \underline{\underline{a^2+2a^3+a^4}} \checkmark$$

$$b) (3b^3+4b^4) \cdot (b^3-b^2) = 3b^6-3b^5+4b^7-4b^6 = \underline{\underline{4b^7-b^6-3b^5}} \checkmark$$

$$c) (5c-2c^3)(2c^2+3c^4) = 10c^3+15c^5-4c^5-6c^7 = \underline{\underline{10c^3+11c^5-6c^7}}$$

$$13/111) (a^3+2a^2b+2ab^2+b^3)(a^3-2a^2b+2ab^2-b^3) =$$

$$a^6 - 2a^5b + 2a^4b^2 - a^3b^3 + 2a^5b - 4a^4b^2 + 4a^3b^3 - 2a^2b^4 + 2a^4b^2 - 4a^3b^3 + 4a^2b^4 -$$

$$2ab^5 + a^3b^3 - 2a^2b^4 + 2ab^5 - b^6 = \underline{\underline{a^6 - b^6}}$$

$$13/112) (16x^4 - 48x^3y + 108x^2y^2 - 108xy^3 + 81y^4) \cdot (4x^2 + 12xy + 9y^2) =$$

$$64x^6 + 192x^5y + 144x^4y^2 - 192x^4y + 576x^3y^2 - 432x^3y^3 + 432x^4y^2 + 1296x^3y^3$$

$$+ 972x^2y^4 - 432x^3y^3 - 1296x^2y^4 + 972xy^5 + 324x^2y^4 + 972xy^5 + 729y^6$$

$$\underline{\underline{64x^6 + 432x^3y^3 + 729y^6}}$$

$$13/113) (a^8 + a^7b + a^6b^2 + a^2b^6 + ab^7 + b^8)(a^4 - a^3b + ab^3 - b^4) =$$

$$a^{12} - a^{11}b + a^9b^3 - a^{10}b^2 + a^8b^4 - a^7b^5 + a^{10}b^2 - a^9b^3 + a^7b^5 - a^6b^6 + a^6b^6$$

$$- a^5b^7 + a^3b^9 - a^2b^{10} + a^5b^7 - a^4b^8 + a^3b^{10} - a^6b^6 + a^4b^8 - a^3b^9 + ab^{11} - b^{12} =$$

$$\underline{\underline{a^{12} - b^{12}}}$$

$$13/115b) [(a+b)^{m+1} + (a+b)^{m-1}] \cdot (a+b) = \underline{\underline{(a+b)^{m+2} + (a+b)^m}}$$

$$14/126b) \frac{1}{4b^6} + \frac{1}{2b^6} = \frac{1b^2}{4b^8} + \frac{2}{4b^8} = \frac{b^2}{4b^8} \checkmark$$

$$127b) \frac{1}{x^3} + \frac{1-x}{x^4} = \frac{x^3}{x^{12}} + \frac{x^8 - x^9}{x^{12}} = \frac{x^8}{x^{12}} = \frac{x^2}{x^4} = \frac{1}{x^2}$$

$$128b) \frac{4}{x} - \frac{2-x}{x^2} + \frac{1-x^2}{2x^3} = \frac{8x^2}{2x^3} - \frac{4x-2x^2}{2x^3} + \frac{1-x^2}{2x^3} =$$

$$\frac{8x^2 - 4x + 2x^2 + 1 - x^2}{2x^3} = \frac{9x^2 - 4x + 1}{2x^3}$$

$$129b) \frac{2}{a^2} - \frac{a-3}{a^3} - \frac{1+a^2}{a^5} = \frac{2a^3}{a^5} - \frac{a^3-3a^2}{a^5} - \frac{1+a^2}{a^5} =$$

$$\frac{2a^3 - a^3 + 3a^2 - 1 - a^2}{a^5} = \frac{a^3 + 2a^2 - 1}{a^5}$$

$$130b) \frac{x}{6(x+y)^4} - \frac{1}{12(x+y)^3} = \frac{2x}{12(x+y)^4} - \frac{(x+y)}{12(x+y)^4} =$$

$$\frac{2x - (x+y)}{12(x+y)^4} = \frac{x-y}{12(x+y)^4}$$

$$14/131b) \frac{x^2-y^2}{4(x-y)^3} - \frac{y^2}{6(y-x)^3} = \frac{3(x+y)(x-y)}{12(x-y)^3} - \frac{2y^2}{12(y-x)^3} + \frac{2y^2}{12(x-y)^3}$$

$$\frac{3(x+y)(x-y) - 2y^2}{12(x-y)(x-y)(x-y)} = \frac{3(x^2-y^2) - 2y^2}{12(x-y)^3}$$

$$14/131b) \frac{x^2-y^2}{4(x-y)^3} - \frac{y^2}{6(y-x)^3} = \frac{3(x^2-y^2)}{12(x-y)^3} - \frac{2y^2}{-12(x-y)^3} =$$

$$\frac{3x^2-3y^2}{12(x-y)^3} + \frac{2y^2}{12(x-y)^3} = \frac{3x^2-y^2}{12(x-y)^3} \checkmark$$

$$14/127b) \frac{1}{x^3} + \frac{1-x}{x^4} = \frac{x}{x^4} + \frac{1-x}{x^4} = \frac{1}{x^4} \checkmark$$

$$14/132) \frac{x(1+y)}{x^n} + \frac{x-y}{x^{n-1}} - \frac{1}{x^{n-2}} = \frac{x+xy}{x^n} + \frac{x(x-y)}{x^n} - \frac{x^2}{x^n} =$$

$$\frac{x+xy+x^2-xy-x^2}{x^n} = \frac{x}{x^n} = \frac{1}{x^{n-1}} \checkmark$$

$$14/133b) \frac{x^{2a}-y^{2b}}{x^{2a}+y^{2b}} + \frac{x^{2a}+y^{2b}}{x^{2a}-y^{2b}} = \frac{(x^{2a}-y^{2b})^2}{(x^{2a}+y^{2b})(x^{2a}-y^{2b})} + \frac{(x^{2a}+y^{2b})^2}{(x^{2a}+y^{2b})(x^{2a}-y^{2b})} =$$

$$\frac{(x^{2a}-y^{2b})^2 + (x^{2a}+y^{2b})^2}{(x^{2a}+y^{2b})(x^{2a}-y^{2b})} = \frac{(x^{4a}-2x^{2a}y^{2b}+y^{4b}) + (x^{4a}+2x^{2a}y^{2b}+y^{4b})}{x^{4a}-y^{4b}} =$$

$$\frac{2x^{4a}+2y^{4b}}{x^{4a}-y^{4b}} = \frac{2(x^{4a}+y^{4b})}{x^{4a}-y^{4b}} \checkmark$$

$$14/133e) \frac{a^2+1}{2a-1} - \frac{a}{2} = \frac{(a^2+1-2a)(2a-1)}{2(2a-1)(2a-1)} = \frac{a^2-a+1}{2a-3} \checkmark$$

14/133f)

$$\frac{1}{(a+b)^2} \left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{a+b} \left(\frac{1}{a} + \frac{1}{b} \right) \right] = \frac{1}{(a+b)^2} \left[\frac{b^2(a+b)}{a^2b^2(a+b)} + \frac{a^2(a+b)}{a^2b^2(a+b)} + \frac{2a^2b^2}{(a+b)a^2b^2} \left(\frac{b}{ab} + \frac{a}{ab} \right) \right] =$$

$$\frac{1}{(a+b)^2} \left[\frac{(a^2+b^2)(a+b)^2 + 2a^2b^2}{a^2b^2(a+b)} \cdot \frac{(a+b)}{(ab)} \right] =$$

$$\frac{1}{(a+b)^2} \cdot \frac{(a^2+b^2)(a+b)^3 + 2a^2b^2(a+b)}{a^3b^3(a+b)} = \frac{(a^2+b^2)(a^2+2ab+b^2) + 2a^2b^2}{a^3b^3(a+b)} =$$

$$\frac{a^4+2a^3b+a^2b^2+a^2b^2+2ab^3+b^4+2a^2b^2}{a^3b^3(a^2+a^2b+ab^2+b^2)} =$$

$$= \frac{(a^2+b^2)(a+b)^2 + 2a^2b^2}{a^3b^3(a+b)} \cdot \frac{1}{a^2b^2}$$

15/134*) Primfaktorenzerlegung von:

a) $250047 : 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7 = 3^6 \cdot 7^3$

b) $20449 : 13 \cdot 11 \cdot 13 \cdot 11 = 13^2 \cdot 11^2$

c) $30625 : 5 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 = 5^4 \cdot 7^2$

14/133f)
$$\frac{1}{(a+b)^2} \left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{a+b} \left(\frac{a+b}{a} + \frac{a+b}{b} \right) \right] = \frac{1}{(a+b)^2} \left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{2(a+b)}{(a+b)ab} \right] =$$

$$\frac{1}{(a+b)^2} \cdot \frac{b^2 + a^2 + 2ab}{a^2 b^2} = \frac{(a+b)^2}{(a+b)^2 \cdot a^2 b^2} = \frac{1}{a^2 b^2}$$

14/133e)
$$\left(\frac{a^2+1}{2a-1} - \frac{a}{2} \right) \cdot \frac{a+2}{2a^2-1} = \frac{(a^2+1-a)(2a-1)}{(2a-1) \cdot 2(a+2)} = \frac{2a^3 - a^2 + 2a - 1 - 2a^2 + a}{2a^3 + 4a - 3a - 6} = \frac{2a^3 - 3a^2 + 3a - 1}{2a^3 + a - 6}$$

$$\left(\frac{a^2+1}{2a-1} - \frac{a}{2} \right) \cdot \frac{2a-1}{a+2} = \frac{a^2+1-a}{-2a+4}$$

16/151b)
$$\frac{(ax)^{2p+5}}{(ax)^{p+4}} = \underline{\underline{(ax)^{p+1}}}$$

16/152a) $40a^5b^6 : 8a^3b^2 = \underline{\underline{5a^2b^4}}$

c) $144x^{13}y^{16} : 12x^8y^9 = \underline{\underline{12x^5y^7}}$

16/153b) $5a^4x^8y^5 : (-5^2x^6y^3) = 625x^2y^2 : (-25x^6y^3) = \underline{\underline{-5^2x^2y^2}}$

16/154b) $\frac{4}{5}a^3b^2c^4 : 0.8a^3b^2c^2 = \underline{\underline{c^2}}$

16/155b) $\frac{24x^3y^4}{25z^6} : \frac{8xy^2}{5z^4} = \frac{24x^3y^4}{25z^6} \cdot \frac{5z^4}{8xy^2} = \frac{3x^2y^2}{5z^2}$

16/156a) $\frac{2a}{3}a^{m+2} : \frac{3b}{4}a^{m-1} = \underline{\underline{\frac{8a}{9b}a^3}}$

16/157a) $\frac{m}{n} \left(\frac{a}{b} \right)^{2n+2} : \frac{n}{m} \left(\frac{a}{b} \right)^{2n} = \underline{\underline{\frac{m^2}{n^2} \left(\frac{a}{b} \right)^2}}$

16/157c) $\left(\frac{ma}{nb} \right)^{2n+1} : \left(-\frac{ma}{nb} \right)^{2n} = \underline{\underline{\frac{ma}{nb}}}$

$$16/158a) \frac{(a+b)^6}{(a+b)^2} = \underline{\underline{(a+b)^4}}$$

$$16/158c) \frac{(p+q)^{2m+5}}{(p+q)^{2m+2}} = \underline{\underline{(p+q)^3}}$$

$$16/159b) \frac{a^3(x-y)^{3n+2}}{a^4(x-y)^{5n+3}} = \frac{1}{\underline{\underline{a(x-y)^{2n+1}}}}$$

$$16/159d) \frac{(a+b)^n \cdot b+a}{a-b} \cdot \frac{(a+b)^{n+2}}{b-a} = \frac{(a+b)^2 \cdot b+a}{b-a} = \underline{\underline{-\frac{(a+b)^3}{a-b}}}$$

$$16/160a) (a^{10}+a^8-a^6) : a^5 = a^5+a^3-a = \underline{\underline{a(a^4+a^3-1)}}$$

$$16/160c) (48m^6+36m^5) : 12m^4 = 4m^2+3m = \underline{\underline{m(4m+3)}}$$

$$16/161a) (12 \cdot 1y^7 + 7 \cdot 7y^5) : 1 \cdot 1y^3 = 11y^4 + 7y^2 = \underline{\underline{y^2(11y^2+7)}}$$

$$16/162a) (a^4+6a^2b+9b^2) : (a^2+3b) = \underline{\underline{a^2+3b}}$$

$$16/163) \frac{a^7-6a^6b^3+14a^5b^6-12a^4b^9}{a^3-2a^2b^3} = \frac{a^4(a^3-6a^2b^3+14ab^6-12b^9)}{a^2(a-2b^3)} =$$

$$\frac{a^2(a^3-6a^2b^3+14ab^6-12b^9)}{a-2b^3}$$

$$16/165) \frac{144a^4-289a^2b^2+100b^4}{12a^2+7ab-10b^2} =$$

$$144a^4-289a^2b^2+100b^4 : 12a^2+7ab-10b^2 = 12a^2-24 \cdot 08\bar{3} \cdot b^2+8 \cdot 3\bar{3} \cdot b^4-2$$

$$\frac{144}{7}a^3-b-17ab+\frac{100}{7}b^3-a-14 \cdot 4a^4-b^2+28 \cdot 9a^2-10b^2$$

$$16/163) (a^7 - 6a^6b^3 + 14a^5b^6 - 12a^4b^9) : (3a^3b - 2a^2b^3) = \underline{a^4 + 4a^3b^3 + 22a^2b^6 + 32ab^9}$$

$$\begin{array}{r} 4a^6b^3 + 14a^5b^6 - 12a^4b^9 \\ -4a^6b^3 + 8a^5b^6 \\ \hline \end{array}$$

$$\begin{array}{r} 22a^5b^6 - 12a^4b^9 \\ 22a^5b^6 + 44a^4b^9 \\ \hline 32a^4b^9 \\ -32a^4b^9 \\ \hline 0 \end{array}$$

$$16/164) (15a^6b^4 - 2a^5b^5 - 24a^4b^6) : (3a^2b - 4ab^2) = \underline{5a^4b^3 + 6a^3b^4} \quad \checkmark$$

$$\begin{array}{r} -15a^6b^4 + 20a^5b^5 \\ 18a^5b^5 - 24a^4b^6 \\ -18a^5b^5 + 24a^4b^6 \\ \hline 0 \end{array}$$

$$16/165) (144a^4 - 289a^2b^2 + 100b^4) : (12a^2 + 7ab - 10b^2) = \underline{12a^2 - 7ab + 10b^2} \quad \checkmark$$

$$\begin{array}{r} -144a^4 + 84a^3b - 120a^2b^2 \\ -289a \end{array}$$

$$\begin{array}{r} -144a^4 + 120a^2b^2 + 84a^3b \\ -169a^2b^2 + 100b^4 - 84a^3b \\ \hline 49a^2b^2 - 70ab + 84a^3b \\ +144a^4 - 84a^3b - 169a^2b^2 + 100b^4 \\ +84a^3b + 49a^2b^2 + 70ab + 100b^4 \\ \hline 120a^2b^2 + 170ab + 100b^4 \\ -120a^2b^2 - 70ab + 100b^4 \end{array}$$

$$16/166) (12x^4 - x^3y - 32x^2y^2 + xy^3 + 20y^4) : (4x^2 + xy - 5y^2) = \underline{3x^2 - xy - 4y^2} \quad \checkmark$$

$$\begin{array}{r} -12x^4 + 3x^3y + 15x^2y^2 \\ -4x^3y - 17x^2y^2 + xy^3 + 20y^4 \\ +4x^3y + x^2y^2 + 5xy^3 \\ \hline -16x^2y^2 - 4xy^3 + 20y^4 \\ +16x^2y^2 + 4xy^3 - 20y^4 \\ \hline 0 \end{array}$$

$$16/167b) (x^{12} + y^{12}) : (x^4 + y^4) = x^8 + \left(\frac{y}{x}\right)^4$$

$$(x^{12} + y^{12}) : (x^4 + y^4) = x^8 - x^4y + y^5$$

$$\begin{array}{r} -x^{12} + x^8y + y^{12} \\ -x^8y - y^{12} \\ \hline +x^8y + x^4y^5 + y^{12} \\ x^4y^5 + y^{12} \\ -x^4y^5 - y^9 + y^{12} \\ \hline y^9 - y^{12} \end{array}$$

$$16/163) (a^7 - 6a^6b^3 + 14a^5b^6 - 12a^4b^9) : (a^3 - 2a^2b^3) = \underline{a^4 + a^3b^3 + 22a^2b^6 + 32a^4b^9}$$

$$-a^7 + 2a^6b^3$$

$$4a^6b^3 + 14a^5b^6 - 12a^4b^9$$

$$-4a^6b^3 + 8a^5b^6$$

$$22a^5b^6 - 12a^4b^9$$

$$-22a^5b^6 + 44a^4b^9$$

$$56a^4b^9$$

$$32$$

$$17/167b) (x^{12} + y^{12}) : (x^4 + y^4) = \underline{x^8 - x^4y^4 + y^8}$$

$$-x^{12} + x^8y^4 + y^{12}$$

$$-x^8y^4 + y^{12}$$

$$-(-x^8y^4 - x^4y^8)$$

$$+x^4y^8 + y^{12}$$

$$-(+x^4y^8 + y^{12})$$

$$0$$

$$17/168b) (a^8 - b^8) : (a^3 + a^2b + ab^2 + b^3) = \underline{a^5 - a^4b + ab^4 - b^5}$$

$$-(a^8 + a^7b + a^6b^2 + a^5b^3)$$

$$-a^7b - a^6b^2 - a^5b^3 - b^8$$

$$-(a^7b - a^6b^2 - a^5b^3 - a^4b^4)$$

$$+a^4b^4 - b^8$$

$$-(a^4b^4 + a^3b^5 + a^2b^6 + ab^7)$$

$$= a^3b^5 + a^2b^6$$

$$-a^3b^5 - a^2b^6 - ab^7 - b^8$$

$$-(a^3b^5 - a^2b^6 - ab^7 - b^8)$$

$$0$$

$$17/169) (9a^8 - 10a^4b^4 + b^8) : (3a^4 + 4a^2b^2 + b^4) = \underline{3a^4 + 4a^2b^2 + b^4}$$

$$-(9a^8 + 12a^6b^2 + 3a^4b^4)$$

$$-12a^6b^2 - 13a^4b^4 + b^8$$

$$-(12a^6b^2 + 16a^4b^4 + 4a^2b^6)$$

$$-3a^4b^4 - 4a^2b^6 + b^8$$

$$3a^4b^4 + 4a^2b^6 + b^8$$

$$-(3a^4b^4 + 4a^2b^6 + b^8)$$

$$0$$

$$17/170) (a^4 - b^4) : (a - b) = \underline{a^3 + a^2b + ab^2 + b^3}$$

$$-(a^4 - a^3b)$$

$$a^3b - b^4$$

$$-(a^3b - a^2b^2)$$

$$+a^2b^2 - b^4$$

$$-(a^2b^2 - ab^3)$$

$$ab^3 - b^4$$

$$-(ab^3 - b^4)$$

$$0$$

$$(a^2 - b^2) : (a + b) = \underline{a - b}$$

$$-(a^2 + ab)$$

$$-ab - b^2$$

$$-(-ab - b^2)$$

$$0$$

$$(a^5 + b^5) : (a + b) = \underline{a^4 - a^3b + a^2b^2 - ab^3 + b^4}$$

$$-(a^5 + a^4b)$$

$$-a^4b + b^5$$

$$-(-a^4b - a^3b^2)$$

$$+a^3b^2 + b^5$$

$$-(+a^3b^2 + a^2b^3)$$

$$-a^2b^3 + b^5$$

$$-(-a^2b^3 - ab^4)$$

$$+ab^4 + b^5$$

$$-(ab^4 + b^5)$$

$$-b^4 + b^5$$

$$16/163) (a^7 - 6a^6b^3 + 14a^5b^6 - 12a^4b^9) : (a^3 - 2a^2b^3) = \underline{a^4 + a^3b^3 + 22a^2b^6 + 32a^4b^9}$$

$$-a^7 + 2a^6b^3$$

$$+a^6b^3 + 14a^5b^6 - 12a^4b^9$$

$$-4a^6b^3 + 8a^5b^6$$

$$22a^5b^6 - 12a^4b^9$$

$$-22a^5b^6 + 44a^4b^9$$

$$56a^4b^9$$

$$32$$

$$17/167b) (x^{12} + y^{12}) : (x^4 + y^4) = \underline{x^8 - x^4y^4 + y^8}$$

$$-(x^{12} + x^8y^4 + y^{12})$$

$$-x^8y^4 + y^{12}$$

$$-(-x^8y^4 - x^4y^8)$$

$$+x^4y^8 + y^{12}$$

$$-(+x^4y^8 + y^{12})$$

$$0$$

$$17/168b) (a^8 - b^8) : (a^3 + a^2b + ab^2 + b^3) = \underline{a^5 - a^4b + ab^4 - b^5}$$

$$-(a^8 + a^7b + a^6b^2 + a^5b^3)$$

$$-a^7b - a^6b^2 - a^5b^3 - b^8$$

$$-(a^7b - a^6b^2 - a^5b^3 - a^4b^4)$$

$$+a^4b^4 - b^8$$

$$-(a^4b^4 + a^3b^5 + a^2b^6 + ab^7)$$

$$= a^3b^5 + a^2b^6$$

$$-a^3b^5 - a^2b^6 - ab^7 - b^8$$

$$-(a^3b^5 - a^2b^6 - ab^7 - b^8)$$

$$0$$

$$17/169) (9a^8 - 10a^4b^4 + b^8) : (3a^4 + 4a^2b^2 + b^4) = \underline{3a^4 + 4a^2b^2 + b^4}$$

$$-(9a^8 + 12a^6b^2 + 3a^4b^4)$$

$$-12a^6b^2 - 3a^4b^4 + b^8$$

$$-(12a^6b^2 + 16a^4b^4 + 4a^2b^6)$$

$$-3a^4b^4 - 4a^2b^6 + b^8$$

$$3a^4b^4 + 4a^2b^6 + b^8$$

$$-(3a^4b^4 + 4a^2b^6 + b^8)$$

$$0$$

$$17/170) (a^4 - b^4) : (a - b) = \underline{a^3 + a^2b + ab^2 + b^3}$$

$$-(a^4 - a^3b)$$

$$a^3b - b^4$$

$$-(a^3b - a^2b^2)$$

$$+a^2b^2 - b^4$$

$$-(a^2b^2 - ab^3)$$

$$ab^3 - b^4$$

$$-(ab^3 - b^4)$$

$$0$$

$$(a^2 - b^2) : (a + b) = \underline{a - b}$$

$$-(a^2 + ab)$$

$$-ab - b^2$$

$$-(-ab - b^2)$$

$$0$$

$$(a^5 + b^5) : (a + b) = a^4 - a^3b + a^2b - ab^2 + b^3$$

$$-(a^5 + a^4b)$$

$$-a^4b + b^5$$

$$-(a^4b - a^3b^2)$$

$$+a^3b^2 + b^5$$

$$-(a^3b^2 + a^2b^3)$$

$$-a^2b^3 + b^5$$

$$-(a^2b^3 - ab^4)$$

$$+ab^4 + b^5$$

$$-(ab^4 + b^5)$$

$$-b^4 + b^5$$

$$0$$

$$\begin{array}{r} (a^2 + b^2) : (a - b) = a + b \\ -(a^2 - ab) \\ \hline ab + b^2 \\ -(ab - b^2) \\ \hline +2b^2 \Rightarrow \text{unlösbar} \end{array}$$

$$|7/171| (a^{m+n} \cdot b^n - 4a^{m+n-1}b^{2n} - 27a^{m+n-2}b^{3n} + 42a^{m+n-3}b^{4n}) : (a^n b^n - 7a^{n-1}b^{2n}) =$$

$$\begin{array}{r} (ab)^{m+2n} - (4ab)^{m+3n-1} - (27ab)^{m+4n-2} + (42ab)^{m+5n-3} : (ab)^n (7ab)^{n-1} = 1^m \cdot \cancel{7^{m+7}} \\ - (ab)^{m+2n} - (7ab)^{m+n+1} - (4ab)^{m+3n-1} - (27ab)^{m+4n-2} + (42ab)^{m+5n-3} \\ - \cancel{(ab)^{m+2n}} + (7ab)^{m+n+1} + (4ab)^{m+3n-1} + (27ab)^{m+4n-2} - (42ab)^{m+5n-3} \end{array}$$

$$\begin{array}{r} = (ab)^{m+2n} - 4ab^{m+3n-1} - 27ab^{m+4n-2} + 42ab^{m+5n-3} : (ab^{2n} - 7ab^{3n-1}) = \underline{\underline{1^m + 3^{m+n-1} - 6^{m+2n-2}}} \\ - (ab)^{m+2n} - 7ab^{m+3n-1} \\ \hline 0 \quad 3ab^{m+3n-1} - 27ab^{m+4n-2} + 42ab^{m+5n-3} \\ - (3ab^{m+3n-1} - 27ab^{m+4n-2}) \\ \hline 0 \quad -6ab^{m+4n-2} + 42ab^{m+5n-3} \\ - (-6ab^{m+4n-2} + 42ab^{m+5n-3}) \\ \hline 0 \end{array}$$

$$|7/173a| (a^3 - b^3)^m : (a^3 - b^3)^m = 1^6$$

$$\begin{array}{r} (a^3 - b^3)^m \\ - (-b^3) \\ \hline 0 \end{array}$$

$$17/173a) (a^9 - b^9)^m : (a^3 - b^3)^m =$$

$$\frac{(a^{9m} - b^{9m}) : (a^{3m} - b^{3m})}{\frac{a^{9m} + (ab)^{9m} - b^{9m}}{(ab)^{9m} - b^{9m}}} = a^{6m} \cdot b^{6m} = \underline{\underline{a^{6m} + b^{6m}}}$$

$$174a) \frac{(x^3 + y^3)^2}{x^3 + x^2y + xy^2} : \frac{(x^2 - xy + y^2)^2}{x^2 - xy + y^2} = \underline{\underline{x + y}} \checkmark$$

$$\frac{x^2y - xy^2 + y^3}{-(x^2y - xy^2 + y^3)}$$

$$17/175a) \left(\frac{1}{2}x^3 - \frac{1}{12}x^2y^2 + \frac{17}{24}xy^4 + \frac{3y^8}{8x} \right) : \left(\frac{3}{4}x^2 - \frac{1}{2}xy^2 + \frac{3}{4}y^4 \right) = \left(\frac{2}{3}x + \frac{4}{9}y^2 + \frac{2}{18} \right)$$

$$- \left(\frac{1}{2}x^3 - \frac{1}{3}x^2y^2 + \frac{1}{2}xy^4 \right)$$

$$\frac{\frac{1}{4}x^2y^2 + \frac{15}{24}xy^4 - \frac{1}{12}x^2y^2 + \frac{17}{24}xy^4 + \frac{3y^8}{8x}}$$

$$- \left(\frac{1}{3}x^2y^2 - \frac{2}{9}xy^4 + \frac{1}{3}y^6 \right)$$

$$\frac{\frac{2}{9}xy^4 - \frac{1}{3}y^6 - \frac{1}{2}xy^4 - \frac{1}{12}x^2y^2 + \frac{17}{24}xy^4 + \frac{3y^8}{8x}}$$

$$\frac{\cancel{\frac{8}{36}xy^4} + \frac{18}{36}xy^4 + \dots}{\dots}$$

$$\frac{16}{72}xy^4 - \frac{36}{72}xy^4 + \frac{51}{72}xy^4 + \dots = \frac{31}{72}xy^4$$

$$\left(\frac{1}{2}x^3 - \frac{1}{12}x^2y^2 + \frac{17}{24}xy^4 + \frac{3y^8}{8x} \right) : \left(\frac{3}{4}x^2 - \frac{1}{2}xy^2 + \frac{3}{4}y^4 \right) = \left(\frac{2}{3}x + \frac{1}{3}y^2 + \frac{1}{2} \right)$$

$$- \left(\frac{1}{2}x^3 - \frac{1}{3}x^2y^2 + \frac{1}{2}xy^4 \right)$$

$$- \left(\frac{1}{2}x^3 - \frac{4}{12}x^2y^2 + \frac{12}{24}xy^4 \right)$$

$$\frac{\frac{1}{4}x^2y^2 + \frac{5}{24}xy^4 + \frac{3y^8}{8x}}$$

$$- \left(\frac{1}{4}x^2y^2 - \frac{1}{6}xy^4 + \frac{3}{12}y^6 \right)$$

$$+ \frac{4}{24}xy^4$$

$$\frac{9}{24} = \frac{3}{8}xy^4 + \frac{3y^8}{8x}$$

$$17/175a) \left(\frac{1}{2}x^3 - \frac{1}{12}x^2y^2 + \frac{17}{24}xy^4 + \frac{3y^8}{8x} \right) : \left(\frac{3}{4}x^2 - \frac{1}{2}xy^2 + \frac{3}{4}y^4 \right) = \underline{\underline{\frac{2}{3}x + \frac{1}{3}y^2 + \frac{1}{2} \frac{y^4}{x}}}$$

$$- \left(\frac{1}{2}x^3 - \frac{1}{3}x^2y^2 + \frac{1}{2}xy^4 \right)$$

$$0 \quad \frac{1}{4}x^2y^2 + \frac{5}{24}xy^4 + \frac{3y^8}{8x}$$

$$- \left(\frac{1}{4}x^2y^2 - \frac{1}{6}xy^4 + \frac{1}{4}y^6 \right)$$

$$0 \quad \frac{3}{8}xy^4 - \frac{1}{4}y^6 + \frac{3y^8}{8x}$$

$$- \left(\frac{3}{8}xy^4 - \frac{1}{4}y^6 + \frac{3y^8}{8x} \right)$$

$$0$$

$$17/176) \left(\frac{a^3c^4}{12b} + \frac{3a}{4} + \frac{2ac^2}{9} - \frac{2b}{ac^2} - \frac{36b^2}{a^3c^6} \right) : \left(\frac{abc}{3} + \frac{3b^2}{ac^3} \right) = \frac{a^2c^3}{4b} + \frac{2c}{3b} + \frac{36b \cdot 3}{ac^7}$$

$$- \left(\frac{a^3c^4}{12b} + \frac{3a^2c^3}{4abc^3} \right)$$

$$0 \quad \frac{2ac^2}{9} - \frac{2b}{ac^2} - \frac{36b^2}{a^3c^6}$$

$$- \left(\frac{2ac^2}{9} + \frac{36b^2}{2abc^2} \right)$$

$$0 \quad \frac{36b^2}{a^3c^6}$$

$$- \left(\frac{36b^2 \cdot 3}{3a^5c^6} + \frac{36b^3 \cdot 9}{a^5c^{10}} \right)$$

$$- \frac{324b^3}{a^5c^{10}}$$

$$17/179b) (12a^6 - 12a^4 - 6a^3 + 6a^2 + a - 20) : (0.5a^2 - 0.25a - 5) = \underline{\underline{2.4a^4 + 12a^3 - 0.8a^2 - 0.4a + 4}}$$

$$- (12a^6 - 0.6a^5 - 12a^4)$$

$$0 \quad +0.6a^5 - 0.7a^4 - 6a^3 + 6a^2 + a - 20$$

$$- (0.6a^5 - 0.3a^4 - 6a^3)$$

$$0 \quad -0.4a^4 \quad 0 \quad +6a^2 + a - 20$$

$$- (-0.4a^4 + 0.2a^3 + 4a^2)$$

$$0 \quad -0.2a^3 + 2a^2 + a - 20$$

$$- (-0.2a^3 + 0.1a^2 + 2a)$$

$$0 \quad 2a^2 - a - 20$$

$$- (2a^2 - a - 20)$$

$$0$$

19/179) a) $x^n - 1$ für $n = 2, 3, 4, 5$

$$\cancel{x-1} \frac{(x+1)(x-1)}{(x^2+1)}, (x-1)(x^2+x+1), \frac{(x^2-x+1)(x^2-x+1)}{(x-1)(x+1)(x^2+1)}$$

b) $x^n + 1$ für $n = 3, 5, 7, 9$

$$\frac{(x+1)(x^2-x+1)}{(x+1)(x^4-x^3+x^2-x+1)} \quad \frac{(x+1)(x^2-x^5+x^4-x^3+x^2-1)}{(x+1)(x^8-x^7+x^6-x^5)}$$

18/180) b) $\frac{x^3}{64} - \frac{y^3}{8} = \frac{x^3}{64} - \frac{8y^3}{64} = \frac{(x-8y)^3}{64} = \frac{x-8y}{4}$

d) $27a^3 + 64b^3 = 27(a^3 + 2b^3) = \underline{27(a+2b)^3} = 3$

f) $\frac{64x^3}{125} + \frac{27y^3}{64} = \frac{4096x^3}{8000} + \frac{3375y^3}{8000} = \frac{(4096x+3375y)^3}{8000} = \frac{4096x+3375y}{20}$

1) H. F. T. K. U.

H. F. K. U.

F. o. T.

T. K.

H

K. U.

T, K,

1. "Meiers werden uns heute abend besuchen", kündigt Herr Müller an. "Die ganze Familie, also Herr und Frau Meier nebst ihren drei Söhnen Tim, Kay und Uwe?" fragt Frau Müller bestürzt. Darauf Herr Müller, der keine Gelegenheit vorbegehen lässt, seine Frau zum logischen Denken anzureizen: "Nein, ich will es dir so erklären: Wenn Vater Meier kommt, dann bringt er auch seine Frau mit. Mindestens einer der beiden Söhne Uwe und Kay kommt. Entweder kommt Frau Meier oder Tim. Entweder kommen Tim und Kay oder beide nicht. Und wenn Uwe kommt, dann auch Kay und Herr Meier. So, jetzt weißt du, wer uns heute abend besuchen wird."

Tim und Kay