Algebra 6a MNG II

mit Conto

Algebra   Michael Schulz   6a

Potenzen mit gebrochenen Exponenten

Ist $a \in N$ $a>0$, $p,q = \in \mathbb{Z}$, so wollen wir folgende neue Schreibweise einführen:

Statt $\sqrt[p]{a^q}$ schreiben wir $a^{\frac{q}{p}}$

$$\boxed{\sqrt[p]{a^q} \overset{Def.}{=} a^{\frac{q}{p}}} \qquad a>0$$

Haben diese neuen Potenzen die gleichen Eigenschaften wie die bisherigen?

1) Beh.: $a^{\frac{p}{n}} \cdot a^{\frac{t}{q}} = a^{\frac{p}{n}+\frac{t}{q}} = a^{\frac{pq+tn}{nq}}$ $a>0$

↳ $\sqrt[n]{a^p} \cdot \sqrt[q]{a^t} = \sqrt[qn]{a^{pq}} \cdot \sqrt[qn]{a^{tn}} = \sqrt[qn]{a^{pq} \cdot a^{tn}} = \sqrt[qn]{a^{pq+tn}}$

rechts nach Def.: $\sqrt[nq]{a^{pq+tn}}$

30/65) a) $-8^{\frac{2}{3}}$ $\sqrt[3]{-8^2} = \sqrt[3]{\cancel{4}64} = \underline{\underline{-4}}$

b) $-32^{\frac{4}{5}} = -\sqrt[5]{32^4} = \underline{-16}$

c) $9^{-\frac{3}{2}} = 9^{-1.5} = 0.037$ $9^{-\frac{2}{3}} = \sqrt[3]{9^{-2}} = \sqrt[3]{\frac{1}{81}} = \underline{\frac{1}{9}}$ $\frac{1}{27}$

d) $27^{-\frac{1}{3}} = \sqrt[3]{27^{-1}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

e) $81^{-\frac{3}{4}} = \sqrt[4]{81^{-3}} = \sqrt[4]{\frac{1}{81^3}} = \sqrt[4]{-}$ $\frac{1}{27}$

f) $(-125)^{-\frac{2}{3}} = \sqrt[3]{-125^{-2}} = \sqrt[3]{\frac{1}{15625}} = \frac{1}{25}$ $\frac{1}{25}$

g) $(-27)^{-\frac{4}{3}} = \sqrt[3]{-27^{-4}} = \sqrt[3]{\frac{1}{27^4}} = \frac{1}{81}$ $\frac{1}{81}$

h) $(-32)^{-\frac{2}{5}} = \sqrt[5]{-32^{-2}} = \sqrt[5]{\frac{1}{1024}} = \frac{1}{4}$ $\frac{1}{4}$

i) $-64^{-\frac{5}{6}} = -\sqrt[6]{64^{-5}} = \sqrt[6]{\frac{1}{64^5}} = -\frac{1}{32}$ $-\frac{1}{32}$

k) $-(-27)^{-\frac{2}{3}} = -\sqrt[3]{-27^{-2}} =$ $-\frac{1}{9}$

2. Beh.: $a^{\frac{p}{q}} : a^{\frac{m}{n}} = a^{\frac{p}{q}-\frac{m}{n}}$ $(a>0)$

$= a^{\frac{pn-qm}{nq}}$

Zum Bew. potenzieren wir beide Seiten mit qn

links $\left(\frac{a^{\frac{p}{q}}}{a^{\frac{m}{n}}}\right)^{qn}$ $\frac{\left(a^{\frac{p}{q}}\right)^{qn}}{\left(a^{\frac{m}{n}}\right)^{qn}} = \frac{a^{pn}}{a^{qm}}$ $\underline{\underline{a^{pn-qm}}}$ rechts: $\left(a^{\frac{pn-qm}{qn}}\right)^{nq} = \underline{\underline{a^{pn-qm}}}$

3. $(a^n)^k = a^{nk}$ $\left(a^{\frac{p}{q}}\right)^{\frac{r}{s}} = a^{\frac{pr}{qs}}$

Beweis: Wir potenzieren beide Seiten mit qs.

links $\left(\left(a^{\frac{p}{q}}\right)^{\frac{r}{s}}\right)^{qs} = \left(a^{\frac{p}{q}}\right)^{rq} = \underline{\underline{a^{pr}}}$ rechts $\left(a^{\frac{pr}{qs}}\right)^{qs} = \underline{\underline{a^{pr}}}$

4. $a^{\frac{p}{q}} \cdot b^{\frac{p}{q}} = (ab)^{\frac{p}{q}}$

Bew.: Die linke Seite bedeutet eigentlich: $\sqrt[q]{a^p} \cdot \sqrt[q]{b^p} = \sqrt[q]{a^p \cdot b^p} = \sqrt[q]{(ab)^p} = (ab)^{\frac{p}{q}}$

q.e.d.

5. $a^{\frac{p}{q}} : b^{\frac{p}{q}} = (a:b)^{\frac{p}{q}}$ $\frac{a^{\frac{p}{q}}}{b^{\frac{p}{q}}} = \left(\frac{a}{b}\right)^{\frac{p}{q}}$

Die linke Seite bedeutet nach Def.: $\frac{\sqrt[q]{a^p}}{\sqrt[q]{b^p}} = \sqrt[q]{\frac{a^p}{b^p}} = \sqrt[q]{\left(\frac{a}{b}\right)^p} = \left(\frac{a}{b}\right)^{\frac{p}{q}}$

q.e.d.

39/182a) $(a+b):(\sqrt[3]{a}+\sqrt[3]{b}) = (a^1+b^1):(a^{\frac{1}{3}}+b^{\frac{1}{3}}) = \underline{\underline{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}}}$

$-(a^1 + a^{\frac{2}{3}}b^{\frac{1}{3}})$

$-a^{\frac{2}{3}}b^{\frac{1}{3}} + b^1$

$-(-a^{\frac{2}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}})$

$a^{\frac{1}{3}}b^{\frac{2}{3}} + b^1$

$-(a^{\frac{1}{3}}b^{\frac{2}{3}} + b^1)$

0

180, 183

39/180a) $(\sqrt{ac} - \sqrt{bc} + \sqrt{ad} - \sqrt{bd}):(\sqrt{a} - \sqrt{b}) = \underline{\underline{\sqrt{c} + \sqrt{d}}}$

$-(\sqrt{ac} - \sqrt{bc})$ $\sqrt{ad} - \sqrt{bd}$

0 0

b) $(x - 6\sqrt{xy} + 5y):(\sqrt{x} - \sqrt{y}) = \sqrt{x} \cancel{+7\sqrt{y}} - 5\sqrt{y} =$

$\underline{\underline{\sqrt{x} - 5\sqrt{y}}}$

$\cancel{-(x - \sqrt{xy})}$

$\cancel{7\sqrt{xy} + 5y}$

$\cancel{-(7\sqrt{xy} - 7y)}$

$\cancel{12y}$

$-(x - \sqrt{xy})$

$-5\sqrt{xy} + 5y$

$(-5\sqrt{xy} + 5y)$

0

39/183 a) $(\sqrt[3]{a^2}-\sqrt[3]{b^2}):(\sqrt[3]{a}+\sqrt[3]{b})=\sqrt[3]{a}-\sqrt[3]{b}$

$-(\sqrt[3]{a^2}+\sqrt[3]{ab})$

$-\sqrt[3]{ab}-\sqrt[3]{b^2}$

$-(-\sqrt[3]{ab}+\sqrt[3]{b^2})$

0

b) $(\sqrt[4]{a^3}-\sqrt[4]{b^3}):(\sqrt[4]{a}-\sqrt[4]{b})=\sqrt[4]{a^2}+\cancel{\sqrt[4]{ab}}+\sqrt[4]{b^2}$

$\sqrt[2]{a}+\sqrt[4]{ab}+\sqrt[2]{b}$

$-(\sqrt[4]{a^3}-\sqrt[4]{a^2b})$

$\sqrt[4]{a^2b}-\sqrt[4]{b^3}$

$-(\sqrt[4]{a^2b}-\sqrt[4]{ab^2})$

$\sqrt[4]{ab^2}-\sqrt[4]{b^3}$

$-(\sqrt[4]{ab^2}-\sqrt[4]{b^3})$

0

$$\frac{3}{\sqrt{6}}=\frac{3\sqrt{6}}{6}=\frac{\sqrt{6}}{2} \qquad \frac{12}{\sqrt{5}-1}\cdot\frac{\sqrt{5}+1}{\sqrt{5}+1}=\frac{12(\sqrt{5}+1)}{4}=3(\sqrt{5}+1)$$

erweitern

43/234 a) $$\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}\cdot\frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{(\sqrt{3}+\sqrt{2})+\sqrt{5}}=\frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\cancel{5}+2\sqrt{6}-\cancel{5}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{12}+\sqrt{18}+\sqrt{30}}{12}$$

Gleichungen mit Wurzeln

$$x=\sqrt{x^2-2}$$

$$x^2=x^2-2$$

keine Lösung

Quadriert man beide Seiten einer Gleichung, so sind die Lösungen unbedingt zu kontrollieren.

43/1a) $\sqrt[3]{3x}=2$

$3x=8$

$x=\frac{8}{3}$

2a) $\sqrt[3]{x}=\sqrt{x}$

$x^2=x^3$

$x^3-x^2=0$

$x^2(x-1)=0$

$x_1=1$

$x_2=0$

2c) $\sqrt[3]{x}=-\sqrt[5]{x}$

$x^5=-x^3$

$x^5+x^3=0$

$x^3(x^2+1)=0$

$x_1=i$

$x_2=-i$

$x_3=0$

44/17a) $$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=\frac{7}{3} \quad |\cdot\sqrt{1+x}+\sqrt{1-x}$$

$$\frac{(\sqrt{1+x}+\sqrt{1-x})^2}{1+x-(1-x)}=\frac{7}{3}$$

$$3(\sqrt{1+x}+\sqrt{1-x})^2=14x$$

$$3(1+\cancel{x}+2\sqrt{(1+x)(1-x)}+1-\cancel{x})=14x$$

$$6+6\sqrt{1-x^2}=14x$$

$$\sqrt{1-x^2}=\frac{14x-6}{6}$$

$$1-x^2=\frac{196x^2-168x+36}{36}$$

$$\cancel{36}-36x^2=196x^2-168x+\cancel{36}$$

$$232x^2-168x=0$$

$$x(232x-168)=0$$

$x_1=0$ falsch!

$x_2=\frac{168}{232}=\frac{21}{29}$

17b, 19

44/17 b) $\dfrac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}-\sqrt{2-x}}=3$

$\dfrac{(\sqrt{2+x}+\sqrt{2-x})^2}{2+x-\cancel{2}+x} = \cancel{3(\sqrt{2+x}+\sqrt{2-x})}$

$\dfrac{2+\cancel{x}+2\sqrt{(2+x)(2-x)}+2-\cancel{x}}{2x}=3=\dfrac{4+2\sqrt{4-x^2}}{2x}=3$

$4+2\sqrt{4-x^2}=6x \qquad \sqrt{4-x^2}=3x-2$

$4-x^2=9x^2-12x+4$

$-10x^2+12x=0$

$x(10x-12)=0$

$\cancel{x_1=0}$

$\underline{\underline{x=1.2}}$ ✓

44/19 a) $\sqrt{x+3}=\sqrt{x-2}+1$

$\cancel{x}+3=\cancel{x}-2+2\sqrt{x-2}+1$

$4=2\sqrt{x-2}$

$2=\sqrt{x-2}$

$4=x-2$

$\cancel{x=2}$

$\underline{\underline{6=x}}$ ✓

b) $\sqrt{x-2}-\sqrt{x-9}=1$

$x-2-2\sqrt{(x-2)(x-9)}+x-9=1$

$\dfrac{2x-11-1}{2}=\dfrac{2x-12}{2}=x-6=\sqrt{(x-2)(x-9)} \quad |^2$

$\cancel{x^2}-12x+36=\cancel{x^2}-11x+18$

$-x=-18$

$\underline{\underline{x=18}}$ ✓

c) $\sqrt{x-5}=\sqrt{x+7}-2$

$\cancel{x}-5=\cancel{x}+7+4-4\sqrt{x+7}$

$4=\sqrt{x+7}$

$16=x+7$

$\underline{\underline{x=9}}$ ✓

d) $\sqrt{x+13}-\sqrt{x-3}=2$

$x+13+x-3-2\sqrt{(x+13)(x-3)}=4$

$2x+10-4=2\sqrt{x^2+10x-39}$

$x+3=\sqrt{x^2+10x-39}$

$\cancel{x^2}+8x+16=\cancel{x^2}+10x-39$

$55=2x$

$\cancel{x=27.5}$ $\underline{\underline{12}}$

44/19 d) $\sqrt{x+13}-\sqrt{x-3}=2$

$x+13+x-3-2\sqrt{(x+13)(x-3)}=4$

$2x+6=2\sqrt{(x+13)(x-3)}$

$x+3=\sqrt{x^2+10x-39}$

$\cancel{x^2}+6x+9=\cancel{x^2}+10x-39$

$48=4x$

$\underline{x=12}$ ✓

45/27a) $2\sqrt{x-12}+5\sqrt{x-9}=3\sqrt{x+3}$

$$4x-48+25x-\cancel{45}^{225}+20\sqrt{(x-12)(x-9)}=\cancel{3}^{9}x+\cancel{9}^{27}$$

$$\cancel{6x-71=20\sqrt{x^2-21x+108}}$$

$$\cancel{\left(\frac{6x-71}{20}\right)^2=x^2-21x+108}$$

$$4x-48+25x-225+20\sqrt{(x-12)(x-9)}=9x+27$$

$$20x-300=-20\sqrt{x^2-21x+108}$$

$$-x+\cancel{300}^{15}=\sqrt{x^2-21x+108}$$

$$\cancel{x^2}-30x+225=\cancel{x^2}-21x+108$$

$$117=9x$$

$$\underline{\underline{x=13}}$$

45/29a) $\sqrt{4x+3}+\sqrt{9x+14}=\sqrt{4x+7}+\sqrt{9x+8}$

$|^2$

$$4x+3+2\sqrt{(4x+3)(9x+14)}+9x+14=4x+7+2\sqrt{(4x+7)(9x+8)}+9x+8$$

$$2+2\sqrt{36x^2+83x+42}=2\sqrt{36x^2+95x+56}$$

$$1+\sqrt{36x^2+83x+42}=\sqrt{36x^2+95x+56}$$

$$1+2\sqrt{36x^2+83x+42}+\cancel{36x^2}+83x+42=\cancel{36x^2}+95x+56$$

$$2\sqrt{36x^2+83x+42}=12x+13$$

4

$$4(36x^2+83x+42)=144x^2+312x+169$$

$$\cancel{144x^2}+332x+168=\cancel{144x^2}+312x+169$$

$$20x=1$$

$$\underline{\underline{x=0.05}}$$

45/31a)

$$\begin{array}{l|l} \sqrt{x}+\sqrt{y}=15 & + \\ \sqrt{x}-\sqrt{y}=5 & - \end{array}$$

$$2\sqrt{y}=10$$

$$\sqrt{y}=5$$

$$\underline{\underline{y=25}}$$

$$\sqrt{x}+5=15$$

$$\sqrt{x}=10$$

$$\underline{\underline{x=100}}$$

45/33a)

$$\left|\begin{array}{l} 2\sqrt{x+28}=3\sqrt{y+14} \\ \sqrt{x+28}=6\sqrt{y-1} \end{array}\right|$$

$$12\sqrt{y-1}=3\sqrt{y+14}$$

$$4\sqrt{y-1}=\sqrt{y+14}$$

$$16y-16=y+14$$

$$15y=30$$

$$\underline{\underline{y=2}}$$

$$2\sqrt{x+28}=3\sqrt{16}=12$$

$$\sqrt{x+28}=6$$

$$x+28=36$$

$$\underline{\underline{x=8}}$$

18ab, 37ab

44/18a) $$\frac{\sqrt{a+x}+\sqrt{b+x}}{\sqrt{a+x}-\sqrt{b+x}}=\frac{a}{b}$$

$$\frac{\sqrt{a+x}+\sqrt{b+x}}{a}=\frac{a(\sqrt{a+x}-\sqrt{b+x})}{b}$$

$$\frac{a+x+b+x+2\sqrt{(a+x)(b+x)}}{a^2}=\frac{a+x+b+x-2\sqrt{(a+x)(b+x)}}{b^2}$$

$$\frac{a+b+2x+2\sqrt{ab+ax+bx+x^2}}{a^2}=\frac{a+b+2x-2\sqrt{ab+ax+bx+x^2}}{b^2}$$

$$ab^2+b^3+2xb^2+2b^2\sqrt{ab+ax+bx+x^2}=a^3+a^2b+2a^2x-2a^2\sqrt{ab+ax+bx+x^2}$$

$$a^2b^4+ab^5+2ab^4x+2ab^4$$

$$\frac{a^3-3a^2b-ab^2-b^3}{4ab}$$

$$\frac{(\sqrt{a+x}+\sqrt{b+x})^2}{a+x-b-x}=\frac{a}{b}$$

37a) 9, 16

b) 25, 1

b) $\underline{\underline{b}}$

$$b(\sqrt{a+x}+\sqrt{b+x})^2=a^2-ab$$

$$b(a+x+2\sqrt{(a+x)(b+x)}+b+x)=a^2-ab$$

$$ab+b^2+2bx+2b\sqrt{(a+x)(b+x)}=a^2-ab$$

$$2b\sqrt{(a+x)(b+x)}=a^2-2ab-b^2-2bx$$

$$4b^2(ab+ax+bx+x^2)=a^4+4a^2b^2+b^4+4b^2x^2-4a^3b-2a^2b^2-4a^2bx+4ab^3+8ab^3x+4b^3x$$

$$\cancel{4ab^3}+4ab^2x+\cancel{4b^3x}+\cancel{4b^2x^2}=a^4+b^4+2a^2b^2+\cancel{4b^2x^2}-4a^3b-4a^2bx+\cancel{4ab^3}+8ab^2x+\cancel{4b^3x}$$

$$4a^2bx-4ab^2x=a^4+b^4+2a^2b^2-4a^3b$$

$$x(4a^2b-4ab^2)=a^4+2a^2b^2-4a^3b+b^4$$

$$x=\frac{a^4+2a^2b^2-4a^3b+b^4}{4a^2b-4ab^2}$$

$$(a^4-4a^3b+2a^2b^2+b^4):(a-b)=\underline{\underline{a^3-3a^2b-ab^2-b^3}}$$
$$-(a^4-a^3b)$$
$$-3a^3b+2a^2b^2+b^4$$
$$-(-3a^3b+3a^2b^2)$$
$$-a^2b^2+b^4$$
$$(a^2b^2-ab^3)$$
$$-ab^3+b^4$$

$$\underline{\underline{x}}=\frac{a^4+2a^2b^2-4a^3b+b^4}{4ab(a-b)}=\underline{\underline{\frac{a^3-3a^2b-ab^2-b^3}{4ab}}}$$

44/13a) $$3(\sqrt{x}-2)(\sqrt{x}+25)=(5+3\sqrt{x})(3+\sqrt{x})$$

$$3(x+23\sqrt{x}-50)=15+14\sqrt{x}+3x$$

$$\cancel{3x}+69\sqrt{x}-150=15+14\sqrt{x}+\cancel{3x}$$

$$55\sqrt{x}=165$$

$$3025x=27225$$

$$\underline{\underline{x=9}}$$

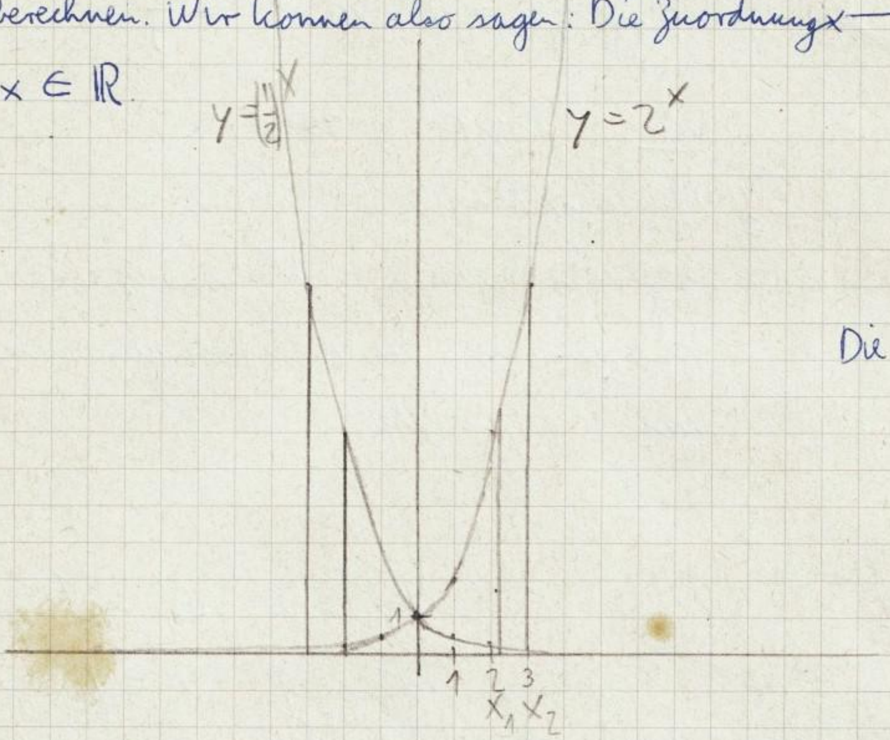
$$12^{0.723} = 12^{\frac{723}{1000}} = \sqrt[1000]{12^{723}}$$

$$x \longrightarrow a^x \quad (a>0)$$

<u>Die Exponentialfunktion</u>

$$x \longrightarrow a^x \ (a>0) \quad x \in \mathbb{R}$$

Für eine positive Grundzahl kann man jede Potenz mit einem reellen Exponenten berechnen. Wir können also sagen: Die Zuordnung $x \longrightarrow a^x$ ist definiert für alle $x \in \mathbb{R}$.



$$2^{-2} = \left(\frac{1}{2}\right)^2$$

$$2^{-3} = \left(\frac{1}{2}\right)^3$$

Die x-Achse wird nie geschnitten

$$2^{1.5} = 2^{\frac{3}{2}} = \sqrt{2^3}$$

$$2^{2.5} = 2^{\frac{5}{2}} = \sqrt{2^5}$$

$$2^{3.5} = 2^{\frac{7}{2}} = \sqrt{2^7}$$

$$2^{1.9} = 2^{\frac{19}{10}} = \sqrt[10]{2^{19}}$$

<u>Eigenschaften dieser Zuordnung:</u> $a^{x_2} > a^{x_1}$ wenn $x_2 > x_1 \land a > 1$

Beweis: aus $a^{x_2} > a^{x_1}$ folgt

$a^{x_2} : a^{x_1} => 1$ und umgekehrt

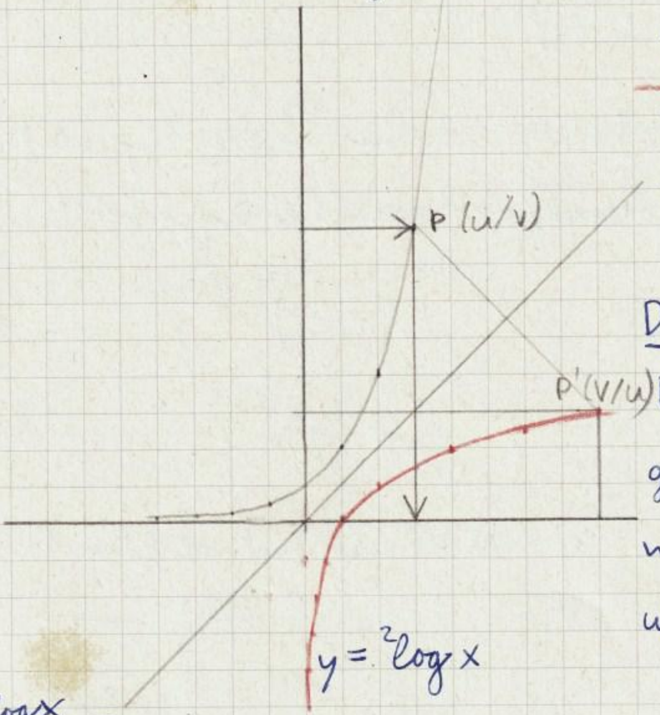
$a^{x_2 - x_1} > 1$ wenn $x_2 > x_1$

Die Werte dieser Funktion steigen monoton wenn $a>1$. Damit ist die Eindeutigkeit der Zuordnung nachgewiesen: $x \longrightarrow a^x$ ist eine Funktion

wenn $a<1$: Kurvensymetrisch

Logarithmen

Weil die Funktion $a^x = y$ $(a > 1)$ monoton steigt, ist sie umkehrbar. Ihre Umkehrfunktion wird Logarithmusfunktion genannt.



Log. v. 32 zur Basis 2

$^{2}\log 32 = 5$ weil $2^5 = 32$

$^{7}\log 343 = 3$ weil $7^3 = 343$

$^{3}\log \frac{1}{9} = -2$ weil $3^{(-2)} = \frac{1}{9}$

Def.: $^{a}\log x = y \Leftrightarrow a^y = x$

Der Logarithmus einer Zahl $x > 0$ zu einer gegebenen Basis $a > 0$ ist derjenige Exponent, mit dem man die Basis potenzieren muss, um x zu erhalten.

$a^{^{a}\log x} = x$

$\log 3 = 0.47712\ldots \quad 10^{0.47712\ldots} = 3$

$^{a}\log a = 1 \quad \text{Inv.}(^{a}\log a) = a$

Inv ln 1 = 2.718… = e (eulersche Zahl)

$\log u = x \quad 10^x = 2^y$

$^{2}\log u = y \quad x = y \cdot \log 2$

$\frac{\log u}{\log 2} = {}^{2}\log u \quad = y = \frac{x}{\log 2}$

$^{2}\log 6 = \frac{\log 6}{\log 2} = 2.5849$

$2^{2.5849}$

Logarithmen zur Basis 2 berechnet man aus 10-erlogarithmen, indem man diese durch den Zehnerlogarithmus von 2 dividiert.

$^{5}\log \frac{1}{\sqrt{125}} = -\frac{3}{2}$ weil $5^{(-\frac{3}{2})} = \sqrt{5^{-3}} = \frac{1}{\sqrt{125}}$

$^{3}\log \frac{1}{\sqrt[5]{81}} = -\frac{4}{5} = -0.8$ weil $3^{-\frac{4}{5}} = (3^{-4})^{\frac{1}{5}} = \frac{1}{\sqrt[5]{81}}$

Logarithmussätze

$^a\log x = u \qquad a^u = x \qquad {}^2\log 32 = 5$

$^a\log y = v \qquad a^v = y \qquad {}^2\log 8 = 3$

$^a\log xy = ? = u+v \qquad {}^2\log 256 = 8$

1) Der Logarithmus eines Produktes zu einer gegebenen Basis ist gleich der Summe der Logrithmen der Faktoren zur gleichen Basis.

In Symbolen: $^a\log xy = {}^a\log x + {}^a\log y$

Beweis: Es sei der $\underbrace{^a\log x = u \quad \text{und} \quad {}^a\log y = v}$

$$a^u = x \qquad a^v = y$$

Der $^a\log xy$ ist z, so dass $a^z = xy$
Andererseits: $xy = a^u \cdot a^v = a^{u+v}$

$a^z = a^{u+v} \Rightarrow z = u+v$

$\log 3 = 0.47712$

$\log 30 = \log(3 \cdot 10) = 1.47712$, da $\log 10 = 1$

$\log 300 = \log(3 \cdot 100) = 2.47712$, da $\log 100 = 2$

2) ~~Der Logarithmus~~

$^a\log \frac{x}{y} = {}^a\log x - {}^a\log y$

Der Logarithmus eines Quotienten zu ~~*~~ irgendeiner Basis ist gleich der Differenz aus dem Logarithmus des Zählers und dem des Nenners zur gleichen Basis.

~~Es seien~~ Es seien: $^a\log x = u$

$^a\log y = v$

$^a\log \frac{x}{y} = z$

Aus der Def. folgt: $a^u = x$, $a^v = y$, $a^z = \frac{x}{y}$

$\frac{x}{y} = \frac{a^u}{a^v} = a^{u-v} \qquad \frac{x}{y} = a^z \longrightarrow a^z = a^{u-v}$

$\Rightarrow z = u - v$

40/201/2/3

40/201 a) $(\sqrt{a+b})^3 \cdot (\sqrt{a+b})^5 = \underline{\underline{(\sqrt{a+b})^8}} = (a+b)^4$

b) $(\sqrt[3]{x+y})^4 \cdot \sqrt[3]{x+y} = \underline{\underline{(\sqrt[\cancel{3}]{x+y})^{\cancel{3}}}} = x+y$

40/202 a) $(\sqrt{a-b})^5 \cdot (\sqrt[4]{a-b})^2 = (\sqrt[\cancel{4}]{(a-b)^{\cancel{2}}}) \cdot (\sqrt[4]{a-b})^{\cancel{2}} = \sqrt{a-b} \cdot \sqrt{a-b} = \underline{\underline{a-b}}$ $(a+b)^3$

b) $(\sqrt[3]{x+y})^5 : (\sqrt[4]{x+y})^3 = (\sqrt[12]{x+y})^{20} : (\sqrt[12]{x+y})^9 = \underline{\underline{(\sqrt[12]{x+y})^{11}}}$ $(x+y)^{\frac{11}{12}}$

40/203 a) $(\sqrt[6]{(a+b)^{\cancel{4}}})^3 + (\sqrt[3]{(a+b)^2})^3 - (\sqrt[4]{(a+b)^2})^6 - (\sqrt[6]{(a-b)^2})^9 =$

$(\sqrt[3]{(a+b)^2})^3 + \sqrt{(a+b)^{\cancel{2}}} - (\sqrt[2]{(a+b)})^{3\cancel{6}} - (\sqrt[\cancel{3}]{(a-b)})^{\cancel{9}3} =$

$\underline{\underline{(a+b)^2 + (a+b)^2 - (a+b)^3 - (a-b)^3}} = 2[(a+b)^2 - a^3 - b]$

~~$a^2+2ab+b^2+a^2+2ab+b^2-(a^3+3a^2b+3ab^2+b^3)-(a^3-3a^2b+3ab^2-b^3) =$~~

$2(-a^3$

b) $(\sqrt[n]{(x+y)^m})^n + (\sqrt[n]{(x-y)^m})^{2n} - (\sqrt[m]{(x+y)^n})^{2m} + (\sqrt[m]{(x+y)^{2n}})^m =$

$(x+y) + (x-y)^{2mn} - (x+y)^{2mn} + (x+y)^{2n}$

$(x+y)^m + (x+y)^{2m}$

$((x+y)^{\frac{m}{n}})^n \to (x+y)^m$

Fortsetzung Logarithmen

3. Den Logarithmus einer Potenz zu irgendeiner Basis bekommt man, wenn man den Logarithmus der Grundzahl mit dem Exponenten multipliziert.

Beisp. $^a\log x^n = n \cdot {}^a\log x$

$\log 5 = 0.6990 \qquad \log 125 = \log 5^3 = 2.0970$

$\log 5^3 = 3 \cdot \log 5$

Beweis: Es sei $^a\log x = u$, $^a\log x^n = v$

Nach Def. des Log gilt: $a^u = x$, $a^v = x^n$

Potenzieren wir beide Seiten v. $a^u = x$ mit n:

$(a^u)^n = x^n$

$a^{nu} = x^n \overset{!}{=} a^v = x^n$

$\Rightarrow a^{nu} = a^v$

d.h. $\underline{\underline{n \cdot u = v}}$ qed.

4) $^{a}\log \sqrt[n]{x} = \frac{1}{n} {}^{a}\log x$

Den log einer Wurzel erhält man, indem man den log des Radikanden durch den Wurzelindex dividiert

Bsp.: $\log 5 = 0.69897$ $\log \sqrt[3]{5} = 0.69897 : 3 = 0.23299$

$^{a}\log \sqrt[y]{x} = \frac{^{a}\log x}{y}$ $\sqrt[3]{5} = 1.7099$

Beweis: Es sei $^{a}\log x = u$, $^{a}\log \sqrt[n]{x} = v$

Nach Def. des log. gilt: $a^{u} = x$, $a^{v} = \sqrt[n]{x}$

Ziehen wir die n-te $\sqrt{\ }$ aus beiden Seiten der Gleichung

$$a^{u} = x$$

es gilt: $a^{\frac{u}{n}} = \sqrt[n]{x} \stackrel{!}{=} a^{v} = \sqrt[n]{x} \Rightarrow a^{\frac{u}{n}} = a^{v}$

$$\frac{u}{n} = v$$

$$7^{x} = 15$$

$$x \log 7 = \log 15$$

$$x = \frac{\log 15}{\log 7}$$

52/32a) $\log 10^{3} = 3 \log 10 = 3$

c $^{2}\log 4^{3} = 6$

e) $^{4}\log 64^{3} = 3 \underbrace{^{4}\log 64}_{3} = \underline{\underline{9}}$

33a) $\lg (xy)^{n} = \underline{\underline{n \lg(xy)}}$

b) $\lg a^{4}b^{5} = \underline{\underline{4 \lg a + 5 \lg b}}$

c) $\lg 25 x^{m} y^{n} = \underline{\underline{2 \lg 5 + m \lg x + n \lg y}}$

d) $\lg [36(a+b)^{5} \cdot (c+d)^{3}] = \underline{\underline{2 \lg 6 + 5 \lg (a+b) + 3 \lg (c+d)}}$

55/66a) $\lg m + \lg n = \underline{\underline{\lg mn}}$

b) $\lg m - \lg n = \underline{\underline{\lg \frac{m}{n}}}$

c) $\lg a + \lg b - (\lg c + \lg d) = \underline{\underline{\lg \frac{ab}{cd}}}$

67a) $\frac{1}{2} \lg x = \lg \sqrt{x}$

c $\frac{m}{n} \lg b = \sqrt[n]{b^{m}}$

Exponentialgleichungen

$2^x = 13$

$x^2 - 13x + 2^{x-1} = 25$

Eine Gleichung, in der die Unbekannte (auch) im Exponenten einer Potenz vorkommt, heisst Exponentialgleichung.

$a^x = a^4 \quad \to a^x = a^3 \to 2^x = 32$

Die einfachste Form der Exponentialgleichung ist: $a^x = b$

Können wir b als eine Potenz von a schreiben, so haben wir Glück.

Im allgemeinen ist dies nicht der Fall. Dann nehmen wir von beiden Seiten den Logarithmus (der Base 10.)

aus $a^x = b$

wird $x \cdot \lg a = \lg b$

$x = \frac{\log b}{\log a}$

Bsp. $2^x = 13$

$x \log 2 = \log 13$

$x = \frac{\log 13}{\log 2} \approx 3.7004397$

Kompliziertere Gleichungen probieren wir, auf die einfachste Form zuzuführen.

Bsp.: $3^{2x} - 5 \cdot 3^x - 36 = 0 \qquad 3^x = y \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y^2 - 5y - 36 = 0$

$(y-9)(y+4) = 0 \qquad y_1 = 9 \longrightarrow 3^x = 9 \Rightarrow \underline{x = 2}$

$y_2 = -4 \qquad 3^x = -4$ keine Lösung

66/1b) $a^{mx} = b$

$x = \frac{\log b}{\log a^m} = \frac{\log b}{m \log a}$

5a) $2^x = \frac{1}{4}$

$2^x = 2^{-2}$

$x = -2$

c) $10^x = 100^{-\frac{3}{2}} = 10^{2(-\frac{3}{2})}$

$x = -3$

6a) $(0.5)^{\frac{3x}{4}} = 64 = 0.5^{-6}$

$\frac{3x}{4} = -6 \qquad x = -8$

b) $\left(\frac{1}{8}\right)^{\frac{3x}{4}} = 16 = \sqrt[4]{\left(\frac{1}{8}\right)^{3x}}$

c) $\left(\frac{1}{5}\right)^{-2x} = 625$

$\underline{x = 2}$

d) $\left(\frac{2}{3}\right)^{-\frac{x}{2}} = \frac{81}{16}$

$\underline{\underline{x = 8}}$

e) $\left(\frac{2}{5}\right)^{-(x+3)} = \frac{125}{8}$

$\left(\frac{5}{2}\right)^{x+3} = \left(\frac{5}{2}\right)^3$

$\underline{\underline{x = 0}}$

f) $\left(\frac{1}{8}\right)^{-\frac{1}{x}} = 32$ $\qquad \left(\frac{1}{8}\right)^{-\frac{5}{3}} = 8^{\frac{5}{3}} = (\sqrt[3]{8})^5 = 2^5 = 32$

$2^{\frac{3}{x}} = 2^5$

$\underline{\underline{x = \frac{3}{5}}}$

67/

17a) $2^x = 100$

$x = \frac{\log 100}{\log 2} \approx 6.64$

b) $10^x = 2.7183$

$x = \frac{\log 2.7183}{\log 10} = 2.7183$ da $\log 10 = 1$

67/

2~~B~~4a) $3^{2x} \cdot 5^{6x-7} = 9^{x-2} \cdot 7^{1-x}$

$2x \log 3 + (6x-7)\log 5 = (x-2)\log 9 + (1-x)\log 7$

$2x \log 3 + 6x \log 5 - 7\log 5 = x\log 9 - 2\log 9 + \log 7 - x\log 7$

$2x\cancel{\log 3} + 6x\log 5 - x\cancel{\log 9} + x\log 7 = 7\log 5 - 2\log 9 + \log 7$

$x(6\log 5 + \log 7) = 7\log 5 + \log 7 - 2\log 9$

$x = \frac{7\log 5 + \log 7 - 2\log 9}{6\log 5 + \log 7}$

$7 \cdot 5\log + \log 7 - 2 \cdot \log 9$

66/8a) $9^x = 3^{x+4}$

$3^{2x} = 3^{x+4}$

$2x = x + 4$

$\underline{\underline{x = 4}}$

b) $16^{2x} = 4^{2(x+3)}$

$4^{4x} = 4^{2x+6}$

$\underset{2}{4}x = \cancel{2x} + 6$

$\underline{\underline{x = 3}}$

c) $25^{\frac{3x}{2}} = 5^{5x-8}$

$5^{2 \cdot \frac{3x}{2}} = 5^{5x-8}$

4

9a) $(4^{3-x})^{2-x} = 1$

$4^{6-5x+x^2} = 1$

$6 - 5x + x^2 = 0$

$(3-x)(2-x) = 0$

$x_1 = 2$

$x_2 = 3$

b) $\sqrt[x]{a} = a^x$

$a^{\frac{1}{x}} = a^x$

$\frac{1}{x} = x$

$x^2 = 1$

$x_{1/2} = 1, -1$

c) $x^{\log x} = 10$

$\log^2 x = 1$

$\log x_1 = 1$

$\log x_2 = -1$

66/10a) $(a^{x-2})^{x+3} = (a^{x+1})^{x-1}$

$a^{x^2+x-6} = a^{x^2-1}$

$x-5 = 0$

$\underline{\underline{x = 5}}$

11a) $10^{(9^x)} = 1000^{(3^x)}$

$10^{(9^x)} = (10^3)^{(3^x)}$

$10^{(9^x)} = 10^{3\cdot 3^x} = 10^{3^{x+1}}$

$9^x = 3^{x+1}$

$(3^2)^x = 3^{x+1}$

$2x = x+1 \quad \underline{\underline{x=1}}$

66/12a) $2^{3x-4}\cdot 4^{2x-3} = 8^{x+2}$

$2^{3x-4}\cdot (2^2)^{2x-3} = (2^3)^{x+2}$

$2^{3x-4}\cdot 2^{4x-6} = 2^{3x+6}$

$2^{7x-10} = 2^{3x+6}$

$7x-10 = 3x+6$

$4x = 16$

$\underline{\underline{x = 4}}$

b) $3^{4x-1}\cdot 9^{2x+1} = 27^x \cdot 3^{5x+1}$

$3^{4x-1}\cdot (3^2)^{2x+1} = (3^3)^x \cdot 3^{5x+1}$

$3^{4x-1}\cdot 3^{4x+2} = 3^{3x}\cdot 3^{5x+1}$

$\cancel{4x+1}\ 3^{8x+1} = 3^{8x+1}$

$\underline{\underline{x = \in \mathbb{R}\text{, beliebig}}}$

c) $7^x + 8\cdot 7^{x-1} = 735$

$7^{x-1}(7+8) = 735 \quad |:15$

$7^{x-1} = 7^2$

$x-1 = 2$

$\underline{\underline{x = 3}}$

d) $4^{x+1} + 16^{x-1} = 1536$

$4^{x+1} + 4^{2x-2} = 1536$

raten! $\underline{\underline{x = 3.5}}$

67/26) $\frac{1}{10}\cdot 2^x = 1000$ $\qquad$ x = Anzahl Falten

$2^x = 10000$ Logarithmieren

$x \log 2 = 4$

$x = \frac{4}{\log 2} \approx 13.3$

27) 100 l

nach 1 Tg. $100\cdot 0.99$ l

" 2 Tg. $100\cdot 0.99^2$ l

" x Tg. $100\cdot 0.99^x = 50$

$100\cdot 0.99^x = 50$

$0.99^x = 0.5$

$x\log 0.99 = \log 0.5$

$\underline{\underline{x}} = \frac{\log 0.5}{\log 0.99} = \underline{\underline{68.967566}}$

1 Fr. nach 1 J. bei 1% Zunahme $= 1\cdot 1.01$

2 J. " " " $= 1\cdot 1.01^2$

11 b, c, 12d, 28

66/11b) $10^{(3^x)} = 10 \cdot 100^{(2^x)} = 10 \cdot 10^{2^{(2^x)}} = 10\,10^{2^x+1}$ 11b)

$\cancel{10^{(3^x)} = (10^3)^{(2^x)} = 10^{2^x+1.5}}$

$\cancel{3x \neq 3^x = 2^{x+1.5}}$

$10^{(3^x)} = 10 \cdot (10^2)^{(2^x)}$

$10^{(3^x)} = 10\,10^{2 \cdot 2^x}$

$10^{(3^x)} = 10^1 \cdot 10^{(2^{x+1})}$

$10^{(3^x)} = 10^{(2^{x+1}+1)}$

$3^x = 2^{x+1}+1$

$\underline{\underline{x = 2}}$

66/13c) $\sqrt[4]{a^{3x+1}} = \sqrt[6]{a^{2x-3}}$

$a^{\frac{3x+1}{4}} = a^{\frac{2x-3}{6}}$

$\frac{3x+1}{4} = \frac{2x-3}{6}$

$9x+3 = 4x-6$

$5x = -9$

$\underline{\underline{x = -\frac{9}{5}}}$

66/12d) $4^{x+1} + 16^{x-1} = 1536$

$4^{x+1} + (4^2)^{(x-1)} = 1536$

$4^{x+1} + (4)^{(x-2)} = 1536 \qquad 4^x = y$

$4^{2x-2} = 4^{2x} \cdot 4^{-2} = \frac{4^{2x}}{16}$

$4y + \frac{y^2}{16} = 1536$

$y^2 + 64y - 16 \cdot 1536 = 0$

$y_{1/2} = \frac{-64 \pm \sqrt{64 \cdot 64 + 64 \cdot 1536}}{2} = \frac{-64 \pm 8\sqrt{64+1536}}{2} = \frac{-64 \pm 320}{2} =$

-32 ± 160

$4^x = 128 \qquad \underline{\underline{x = 3.5}}$

$2^{2x} + 2^7$

67/28) $\underbrace{\underbrace{\underbrace{0.632 \cdot 0.35}\ 0.35}\cdot 0.35}$

$0.632 \cdot 0.35^x = 0.000001g$

$\log 0.632 + x \lg 0.35 = -6$

$x \log 0.35 = -6 - \log 0.632$

$\cancel{x = 2x}$

$x = \frac{-6 - \log 0.632}{\log 0.35} \approx \underline{\underline{12.72}}$

66/14a) $4^{2x}-8\cdot 4^{x}+12=0 \quad 4^x = z \quad (4^x)^2 = 4^{2x}$

$4^{2x}-4^2\cdot 4^x+12=0$

$z^2-8z+12=0 \qquad z_1=6 \quad z_2=2$

$(z-6)(z-2)=0$

$4^x=6$

$x\log 4=\log 6$

$\underline{\underline{x_1=\frac{\log 6}{\log 4}}}=\underline{\underline{1.29248125}}$

$4^x=2$

$\underline{\underline{x_2=0.5}}$

$(x-2)(x+5)(x-7)=0$

$x^3-4x^2-31x+70=0$

↳ Produkt der drei Lösungen!

Probieren, division der ersten Lösung

14 b, c 15 b c

66/15 a) $\left|\begin{array}{l} 2^x\cdot 2^y=64 \\ 3^x : 3^y=81\end{array}\right|$

$\left|\begin{array}{l} 2^{x+y}=2^6 \\ 3^{x-y}=3^4\end{array}\right|$

$\left|\begin{array}{l} x+y=6 \\ x-y=4\end{array}\right| \begin{array}{l} + \\ + \end{array}$

$2y=2$

$y=1$

$2x=10$

$x=5$

b) $\left|\begin{array}{l} 4^x\cdot 4^y=256 \\ 5^x : 5^y=125\end{array}\right|$

$\left|\begin{array}{l} 4^{x+y}=4^4 \\ 5^{x-y}=5^3\end{array}\right|$

$\left|\begin{array}{l} x+y=4 \\ x-y=3\end{array}\right| \begin{array}{l} + + \\ - + \end{array}$

$2y=1$

$\underline{\underline{y=\frac{1}{2}}}$

$2x=7$

$\underline{\underline{x=3\frac{1}{2}}}$

c) $\left|\begin{array}{l} 5^{3x}\cdot 5^y=3125 \\ 5^{6x} : 5^{2y}=25\end{array}\right|$

$\left|\begin{array}{l} 5^{3x+y}=5^5 \\ 5^{6x-2y}=5^2\end{array}\right|$

$\left|\begin{array}{l} 3x+y=5 \\ 6x-2y=2\end{array}\right| \cdot 2$

$\left|\begin{array}{l} 6x+2y=10 \\ 6x-2y=2\end{array}\right| \begin{array}{l} + + \\ - + \end{array}$

$\to 4y=8$

$\underline{y=2}$

$12x=12$

$\underline{\underline{x=1}}$

14b) $2^{2x}+2^{3x}-2^x=76 \quad 2^x=y$

$y^2+y^3-y=76$

$\cancel{(y-y^{-1})(y+y^2)}$

y

$x=2$

$z^3+z^2-z-76=0$

für den Fall $z\neq 4$

$z=4$

$(z^3+z^2-z-76):(z-4)=z^2+5z+19$

$\begin{array}{l} -z^3+4z^2 \\ \hline 5z^2-z-76 \\ -5z^2+20z \\ \hline 19z-76 \\ -19z+76 \\ \hline 0 \quad 0 \end{array}$

$(z-4)(z^2+5z+19)=0$

$2^x=z=4$

$x\log 2=\log 4$

$\underline{\underline{x}}=\frac{\log 4}{\log 2}\underline{\underline{=2}}$

66

67

66/14c) $2^{x+1} + 2^{2(x+1)} - 2^{2x-1} = 240$ $\quad z = 2^{x+1}$

$z + z^2 - 0.125z^2 - 240 = 0$ $\quad x = 3$

$0.875z^2 + z - 240 = 0$

$2^{2x-1} =$ $z^2 \cdot z^{-3} = z^{2x+1} \cdot z^{-3}$

$2^2 + 2^x + \frac{1}{2}(2^2 + 2^x + 2) \cdot \frac{1}{8}$

$2^1 \cdot 2^x = 2^{x+1}$ $\quad 2^x = z$

$2^2 \cdot 2^{2x} = 2^{2x+2}$

$2^{-1} \cdot 2^{2x} = 2^{x-1}$

2^x

$2z + 4z^2 - 0{,}5z^2 = 76$

$3{,}5z^2 + 2z - 76 = 0 <$ $-\frac{60}{7}$, 8 $\quad z = 8 \quad 2^x = 8 = 2^3$ $\quad x = 3$

67/16c) $\begin{vmatrix} x^2 + y^2 = 425 \\ \log x + \log y = 2 \end{vmatrix}$ $\begin{vmatrix} x^2 + y^2 = 425 \\ xy = 100 \end{vmatrix}$ $\quad x^2 + \left(\frac{100}{x}\right)^2 = 425$ $\quad x^2 = z$

$\cancel{x^2 + 100}$

$\log xy = 2$ $\quad y = \frac{100}{x}$ $\quad z + \frac{10000}{z} = 425$

$\to xy = 100$ $\quad z^2 - 425z + 10000 = 0$

$x_1 = 20 \; y_1 = 5$ $\quad (z-25)(z-400) = 0$

$x_2 = 5 \; y_2 = 20$ $\quad z_1 = 25 \; z_2 = 400$

$x_1 = \pm 5 \; x_2 = \pm 20$

$\begin{vmatrix} x^2 + y^2 = 425 \\ xy = 100 \end{vmatrix} \cdot 2 \begin{vmatrix} + \\ + \end{vmatrix}$

$(x+y)^2 = 625$ $\quad x + y = \pm 25$

$(x-y)^2 = 225$ $\quad x - y = \pm 15$

67/24a) $3^{2x} \cdot 5^{6x-7} = 9^{x-2} \cdot 7^{1-x}$

$$2x\log 3 + (6x-7)\log 5 = (x-2)\log 9 + (1-x)\log 7$$

$$2x\log 3 + 6x\log 5 - 7\log 5 = x\log 9 - 2\log 9 + \log 7 - x\log 7$$

$$2x\log 3 + 6x\log 5 - x\log 9 + x\log 7 = 7\log 5 - 2\log 9 + \log 7$$

$$x(\cancel{2\log 3} + 6\log 5 - \cancel{\log 9} + \log 7) = 7\log 5 - 2\log 9 + \log 7$$

$$\underline{\underline{x = \frac{7\log 5 - 2\log 9 + \log 7}{6\log 7 + \log 7}}}$$

67/16b)

$$\left| \begin{array}{l} \sqrt[x]{4} \cdot \sqrt[y]{4} = 2\sqrt[3]{4} \\ \sqrt[x]{12^{12}} : \sqrt[y]{12^{18}} = 1 \end{array} \right|$$

$$\left| \begin{array}{l} 4^{\frac{1}{x}} \cdot 4^{\frac{1}{y}} = 2\sqrt[3]{4} \\ 12^{\frac{12}{x}} : 12^{\frac{18}{y}} = 1 \end{array} \right|$$

$$4^{\frac{1}{x}+\frac{1}{y}} = 2\sqrt[3]{4} = 2(4^{\frac{1}{3}})$$

$$12^{\frac{12}{x}-\frac{18}{y}} = 12^0$$

$$\frac{12}{x} - \frac{18}{y} = \cancel{12}\,0$$

$$12y - 18x = 0$$

$$y = \frac{18x}{12} = \frac{9x}{6} = \frac{3x}{2}$$

$$4^{\frac{1}{\frac{3x}{2}} + \frac{1}{x}} = 2(4^{\frac{1}{3}})$$

$$4^{\frac{2}{3x} + \frac{1}{x}} =$$

$$\sqrt[3x]{4^2} \cdot \sqrt[x]{4} = 2\sqrt[3]{4}$$

$$\log \sqrt[3x]{4^2} \log \sqrt[x]{4} = \log 2\sqrt[3]{4}$$

$$\frac{\log 16}{3x} \cdot \frac{\log 4}{x} = \frac{2\log 4}{3}$$

$$\frac{1}{3}\frac{\log 16}{x} \cdot \frac{\log 4}{x} = \frac{2\log 4}{3}$$

$$\frac{1}{x}\left(\frac{1}{3}\log 16 \cdot \log 4\right) = \frac{2\log 4}{3}$$

$$x = \frac{\log 16 \cdot \log 4}{2\log 4}$$

5g Halbwertszeit von T = 1200 J

Nach welcher Zeit gibt es nur noch $\frac{1}{1000}$ g?

Nach 1 · 1200 J gibt es noch $\frac{1}{2}$ 5g

" 2 · " J " " " $\left(\frac{1}{2}\right)^2$ 5g

" 3 · " J " " " $\left(\frac{1}{2}\right)^3$ 5g

" x · " J " " " $\left(\frac{1}{2}\right)^x$ 5g

x ist die Anzahl der Zeitspannen von je 1200 J.en

$$\left(\frac{1}{2}\right)^x 5\cancel{g} = \frac{1}{1000}$$

$$x\log\frac{1}{2} = \log\frac{1}{5000}$$

$$x = \frac{\log\frac{1}{5000}}{\log\frac{1}{2}} = 12.287712$$

$$\underline{\underline{\approx 15000 \text{ Jahre}}}$$

67/23a) $100^{(4^x)} = 1000^{(2^x)}$

$$(10^2)^{(4^x)} = (10^3)^{(2^x)}$$

$$10^{2(4^x)} = 10^{3(2^x)}$$

$$2(2^{2x}) = 3(2^x)$$

$$2(2^x)^2 = 3(2^x)$$

$$2(2^x) = 3$$

$$2^x = \frac{3}{2}$$

$$\underline{\underline{x = \frac{\log\frac{3}{2}}{\log 2} = 0.5849625}}$$

67/16a)

$$\left|\begin{array}{l} \sqrt[x]{a}\cdot\sqrt[y]{a}=\sqrt[20]{a^9} \\ \sqrt[x]{a^4}:\sqrt[y]{a^5}=1 \end{array}\right|$$

$$\left|\begin{array}{l} a^{\frac{1}{x}+\frac{1}{y}}=a^{\frac{9}{20}} \\ a^{\frac{4}{x}-\frac{5}{y}}=a^{0} \end{array}\right|$$

$$\left|\begin{array}{l} \frac{1}{x}+\frac{1}{y}=\frac{9}{20} \\ \frac{4}{x}-\frac{5}{y}=0 \end{array}\right| \cdot 4$$

$$\left|\begin{array}{l} \frac{4}{x}+\frac{4}{y}=\frac{36}{20} \\ \frac{4}{x}-\frac{5}{y}=0 \end{array}\right| \begin{array}{l} + \\ - \end{array}$$

$$\rightarrow \frac{4}{y}+\frac{5}{y}=\frac{36}{20}$$

$$\frac{9}{y}=\frac{36}{20}$$

$$180=36y$$

$$\underline{\underline{y=5}}$$

$$\left|\begin{array}{l} \frac{1}{x}+\frac{1}{y}=\frac{9}{20} \\ \frac{4}{x}-\frac{5}{y}=0 \end{array}\right| \cdot 5$$

$$\left|\begin{array}{l} \frac{5}{x}+\frac{5}{y}=\frac{45}{20} \\ \frac{4}{x}-\frac{5}{y}=0 \end{array}\right| \begin{array}{l} + \\ + \end{array}$$

$$\frac{9}{x}=\frac{45}{20}$$

$$180=45x$$

$$\underline{\underline{x=4}}$$

66/14c)

$$2^{x+1}+2^{2(x+1)}-2^{2x-1}=240 \qquad 2^{x+1}=z$$

$$z+z^2-(z^2)2^{-3}-240=0$$

$$z^2\underbrace{(1-2^{-3})}_{\frac{7}{8}}+z-240=0$$

$$\frac{7}{8}z^2+z-240=0$$

$$z_{1/2}=\frac{-1\pm\sqrt{1+840}}{2\frac{7}{8}}=\begin{cases}16\\ -17.142857=-17\frac{1}{7}\end{cases}$$

$$z=2^{x+1}$$

$$2^4=2^{x+1}$$

$$4=x+1$$

$$\underline{\underline{x=3}}$$

$$2^{x+1}=-17\frac{1}{7}=-\frac{120}{7}$$ → keine Lösung

$$\cancel{(2^{2x-1})\cdot u=(2^{2x+2})}$$

$$\cancel{u=2^{2x+2}:2^{2x-1}=}$$

$$2^{2(x+1)}\cdot u=2^{2x-1}$$

$$u=2^{2x-1}:2^{2x+2}=\underline{2^{-3}}$$

$$2^{2x+2}\,2^{-3}=2^{2x-1}$$

67/25d) $\cancel{\sqrt[x]{15.9}}$

$$\sqrt[x]{1.7649}=\sqrt[5]{5}$$

$$1.7649^{\frac{1}{x}}=5^{\frac{1}{5}}$$

$$\log 1.7649^{\frac{1}{x}}=\log 5^{\frac{1}{5}}$$

$$\frac{1}{x}\log 1.7649=\log 5^{\frac{1}{5}}$$

$$\underline{\underline{x}}=\frac{\log 1.7649}{\log 5^{\frac{1}{5}}}=\underline{\underline{1.7649}}$$

e)

$$\sqrt{x^{\log\sqrt{x}}}=10$$

$$x^{\log\sqrt{x}}=100$$

$$\log x^{\log\sqrt{x}}=2$$

$$\log\sqrt{x}\,\log x=2$$

$$\log x^{\frac{1}{2}}\log x^{1}=2$$

$$\frac{1}{2}\log^2 x=2$$

$$\log^2 x=4$$

$$\log x=2$$

$$\underline{\underline{x=100}}$$

S.A. Verbesserung

1) $\left(\frac{1}{10}\right)^{2^x} = \left(\frac{1}{100}\right)^{3^x}$ $\qquad$ $\left(\frac{1}{10}\right)^{2^x} = \left(\frac{1}{10}\right)^{2\cdot 3^x}$ $\qquad$ $\log 2 - \log 2 + x\log 3$

$x(\log 2 - \log 3) = \log 2$

$\frac{1}{10} = \left[\left(\frac{1}{10}\right)^2\right]^{3^x}$ $\qquad$ $2^x = 2\cdot 3^x$

$x = \frac{\log 2}{\log 2 - \log 3} = -1.7095113$

2) $3^{2x} - 5\cdot 3^x + 6 = 0$ $\qquad$ $3^x = y$

$y^2 - 5y + 6 = 0$ $\qquad$ $3^x = 3 \Rightarrow x - 1$

$(y-3)(y-2) = 0$ $\qquad$ $3^x = 2 \Rightarrow x - \longrightarrow \frac{\log 2}{\log 3} = 0.63092975$

2b) $2^{4x}\cdot 3^{2x+1}\cdot 5^{6x-1} = 6^{2x}\cdot 10^{4x-1}\cdot 15^{x+0.5}$

$2\frac{1}{2}\cdot \cancel{2^{4x}}\cdot\cancel{3^{2x}}\cdot 3\cdot 5^{6x}\cdot\frac{1}{5} = 2^{2x}\cdot\cancel{3^{2x}}\cdot\cancel{2^{4x-1}}\cdot 5^{4x-1}\cdot 3^{x+0.5}\cdot 5^{x+0.5}$

$2\cdot 3\cdot 5^x = 2^{2x}\cdot 3^{x+0.5}\cdot 5^{0.5}$

$\log 2 + \log 3 + x\log 5 = 2x\log 2 + x\log 3 + 0.5\log 3 + 0.5\log 5$

$x\log 5 - 2\log 2 - \log 3) = 0.5\log 5 - 0.5\log 3\log 2$

$\underline{\underline{x}} = \frac{0.5(\cancel{\log 5 - \log 3 - 2\log 2})}{\cancel{\log 5 - 2\log 2 - \log 3}} = \underline{\underline{0.5}}$

6) $\left|\begin{array}{l} 2^{\sqrt{x}+\sqrt{y}} = 512 \\ \log\sqrt{xy} = 1 + \log 2 \end{array}\right|$ $\qquad$ $\left|\begin{array}{l} \sqrt{x}+\sqrt{y} = 9 \\ \sqrt{xy} = 20 \end{array}\right|$ $\qquad$ $\sqrt{x} + \frac{20}{\sqrt{x}} = 9$

$x + 20 = 9\sqrt{x}$

$xy = 400$ $\qquad$ $x - 9\sqrt{x} + 20 = 0$

$\sqrt{y} = \frac{20}{y}$ $\qquad$ $(\sqrt{x}-4)(\sqrt{x}-5) = 0$

$x_1 = 16$ $\quad$ $x_2\,25$

$y_1 = 25$ $\quad$ $y_2 = 16$

Definitionsbereich Wertebereich

M: 

$x \to y = ax + b$

$x \to ax^2$ für $a = 1$ $x \to x^2$

5

x y

$\underbrace{5-3}_{2}$ $\underbrace{5+2}_{7}$

1 4 9 16 25 36 $\frac{36}{}$ - 49

$\underbrace{3}_{2}$ $\underbrace{5}_{2}$ $\underbrace{7}_{2}$ $\underbrace{9}_{2}$ 11

11 + 2

Die quadratische Funktion

~~allgemeine~~

Wird jedem Element x eines Def.-bereichs durch die Vorschrift

$x \longrightarrow ax^2 + bx + c$ eine Zahl zugeordnet, so redet man von einer quadratischen Funktion.

Der Def.-bereich ist im allgemeinen die Menge der reellen Zahlen, der Wertebereich ebenfalls.

Die Menge der Zahlenpaare $(x \mid ax^2 + bx + c)$ nennt man die Erfüllungsmenge der Funktion.

Bsp.: $x \longrightarrow 2x^2 - x + 1$

(0 | 1) (5 | 46)
(1 | 2) ⋮
(-1 | 4) ⋮

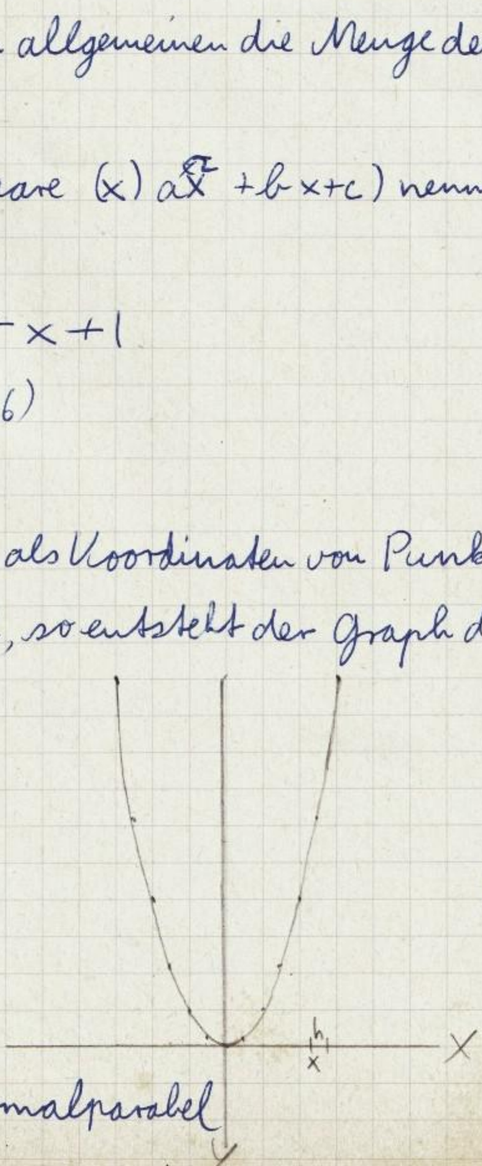
Fasst man diese Zahlenpaare als Koordinaten von Punkten eines rechtwinkligen Koordinatensystems auf, so entsteht der Graph der Funktion.

Beispiele:

$x \longrightarrow x^2$

$y = x^2$

x	y	x	y
0	0	0.5	0.25
1	1	-0.5	0.25
-1	1	1.5	2.25
2	4	-1.5	2.25
-2	4	2.5	6.25
3	9	-2.5	6.25
-3	9		



Normalparabel

Beh.: Die Funktionswerte steigen monoton für $x > 0$, fallen monoton für $x < 0$

Diese Beh. heisst: $(x + h)^2 > x^2$ $(x > 0)$

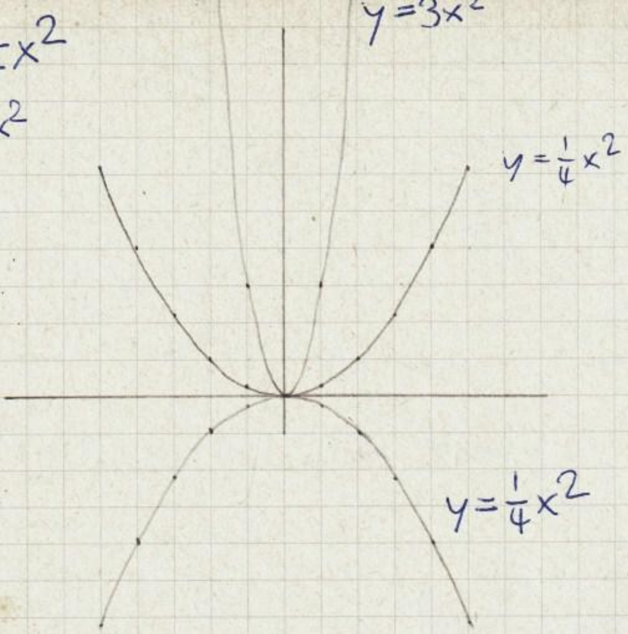
Bew. durch Ausrechnung

$x^2 + \underbrace{2hx + h^2}_{\text{positiv}}$ $x > 0$

$x^2 + \underbrace{2hx + h^2}_{< x^2}$ $x < 0$ $h > 0$

negativ u. im Betrag $> h^2$

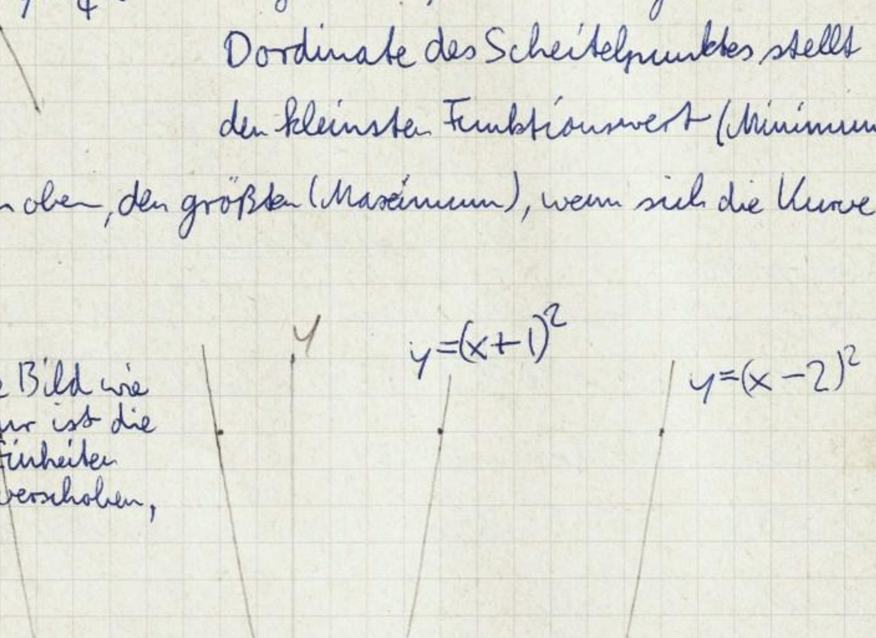
$y = \frac{1}{4}x^2$
$y = 3x^2$



Für die Funktion $y=ax^2$ gilt ausserdem: Der Funktionsgraph öffnet sich nach oben für $a>0$, nach unten für $a<0$. Die Parabel ist schmäler als die Normalparabel, wenn der Betrag v. $a>1$, breiter für $a<1$. Alle Parabeln $y=ax^2$ besitzen die y-Achse als Symetrieachse. Dordinate des Scheitelpunktes stellt den kleinsten Funktionswert (Minimum) dar, wenn sich die Kurve nach oben, den größten (Maximum), wenn sich die Kurve nach unten öffnet.

Bsp.: $y=(x-2)^2$ hat das gleiche Bild wie $y=ax^2$, nur ist die Parabel p Einheiten nach rechts verschoben, wenn $p>0$.

x	y
0	4
1	1
-1	9
2	0
[illegible]	9
-2	16
3	1
4	4
5	9



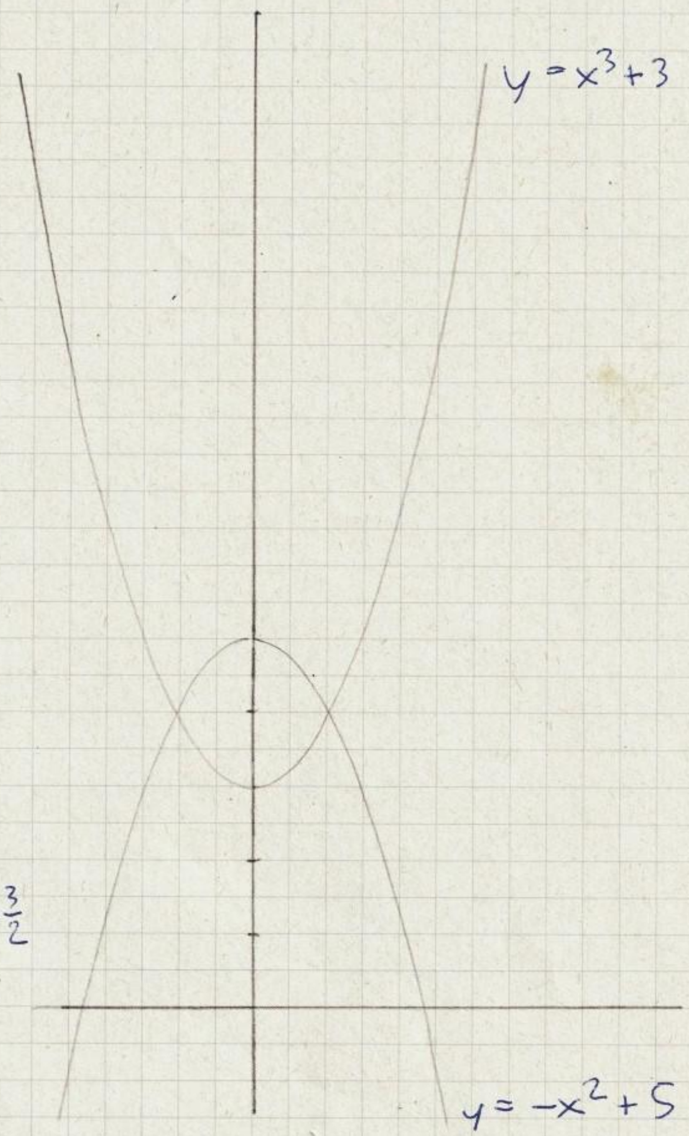
$y=(x+1)^2$

x	y
-4	9
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

$y = ax^2 + q$

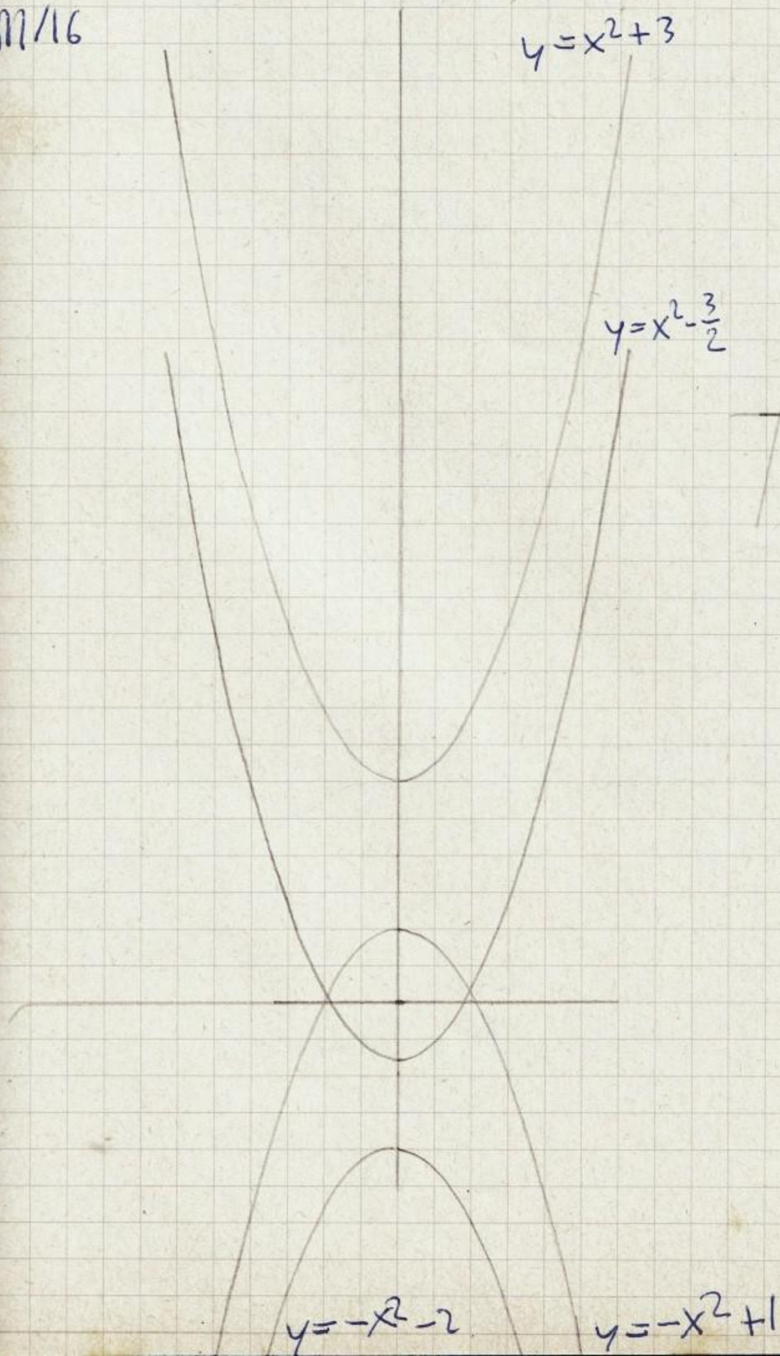
Hier wird q zum Funktionswert $y = ax^2$ dazugezählt. Dies bewirkt eine Parallelverschiebung in Richtung der y-Achse

$y = x^2 + 3$

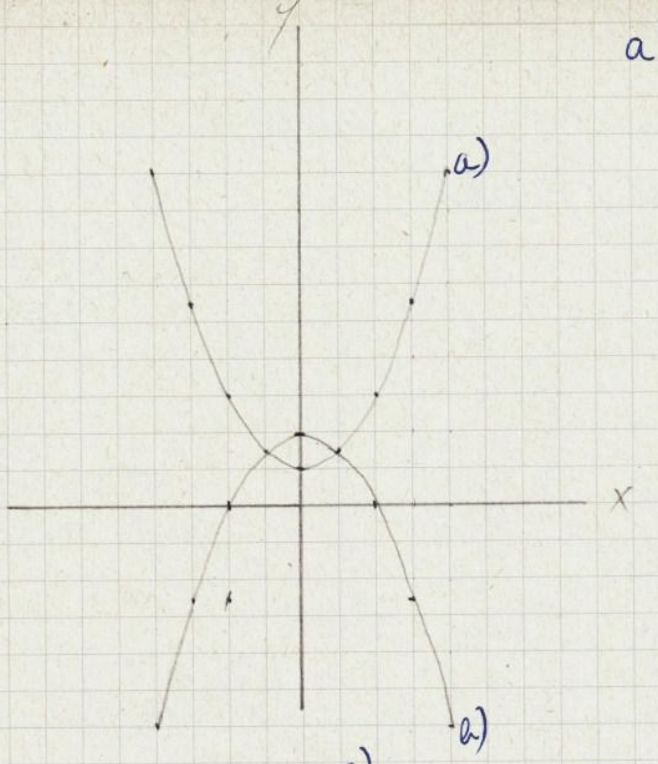


Werden beide Veränderungen vorgenommen, so entsteht die Funktion $y = (a - p)^2 + q$, welche durch eine Parabel dargestellt wird, ~~mit~~ dessen Scheitelpunkt sich in S(p/q) befindet.

M/16



111/16)



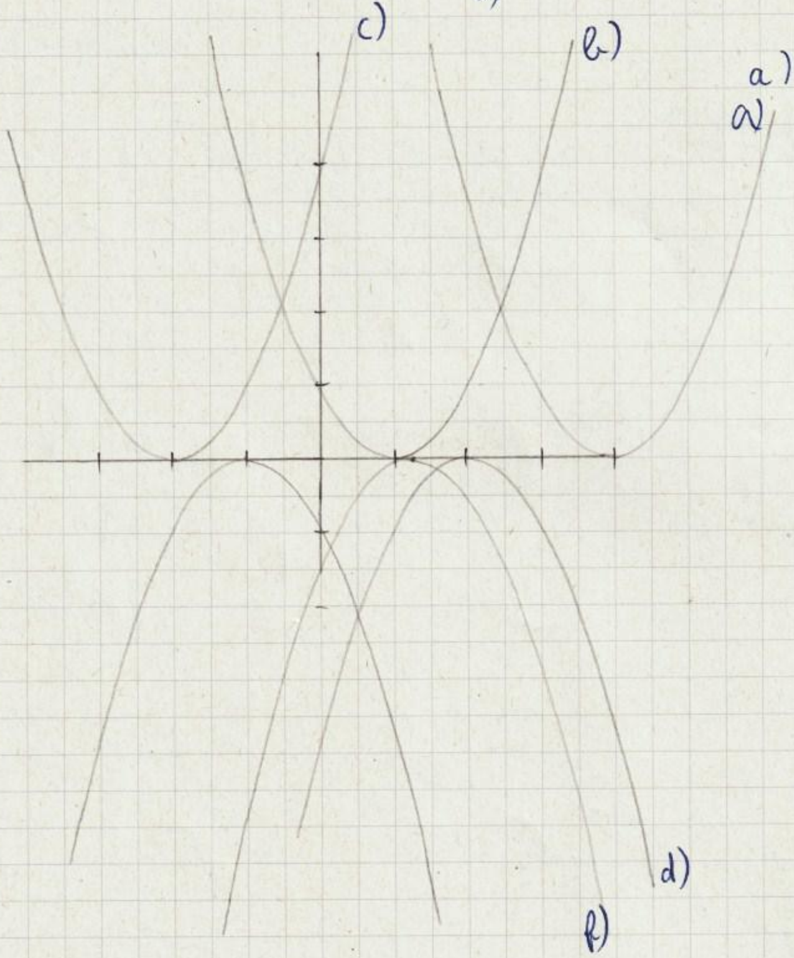
a) $y=\frac{1}{2}x^2+1$

x	y
0	1
1	1.5
2	3
3	5.5
4	9

b) $y=-\frac{1}{2}x^2+2$

x	y
0	2
1	1.5
2	0
3	-2.5
4	-6

111/17)



a) $y=(x-4)^2$ b) $y=(x-1)^2$

c) $y=(x+2)^2$ d) $y=-(x-2)^2$

e) $y=-(x+1)^2$

~~e~~ f) $y=-\left(x\ \frac{3}{2}\right)^2$

111/18) a) $y=x^2-6x+9$ b c d , 20

$y=(x-3)^2$

112 / 20 b) $\cancel{y=\frac{1}{2}(x+1)^2}$

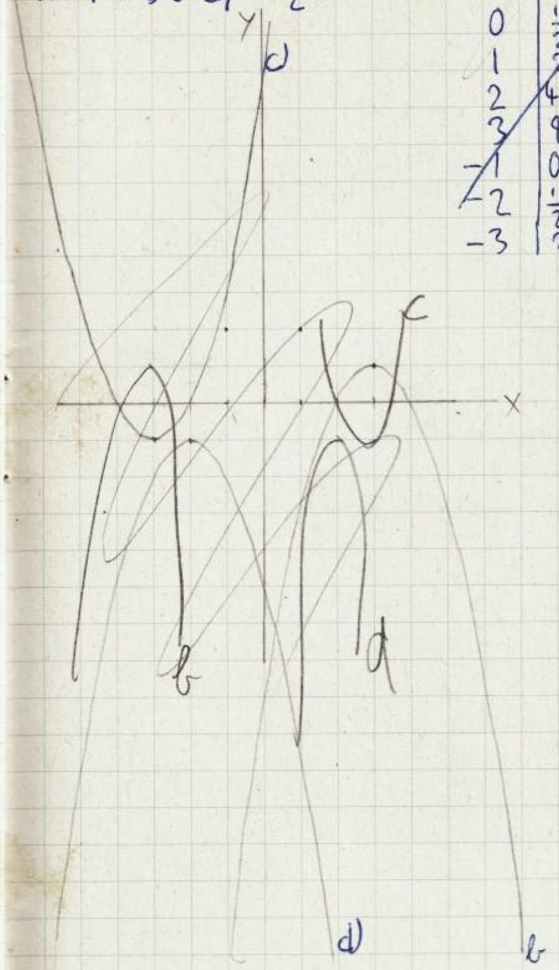
x	y
0	$\frac{1}{2}$
1	2
2	4.5
3	8
-1	0
-2	$\frac{1}{2}$
-3	2

$y=-(x+3)^2+1$

c) $y=(x-3)^2-1$

d) $y=-(x-2)^2-1$

a) $y=(x-1)^2+2$



21 a) $y = x^2-2x+2$
$= x^2-2x+1+1$
$=(x-1)^2+1$

$S=(1/1)$

b) $y=x^2-6x+11$
$=(x-3)^2+2$

$S(3/2)$

c) $y=x^2+4x+5$
$=(x+2)^2+1$

$S(-2/1)$

d) $y=x^2+2x+3$
$(x+1)^2+2$

$S=(-1/2)$

e) $y=x^2-2x$
$=x^2-2x+1-1$
$=(x-1)^2-1$

$S(1/-1)$

f) $y=x^2-4x$
$x^2-4x+4-4$
$(x-2)^2-4$

$S(2/-4)$

g) $y=2x^2-4x+3$
$=2(x^2-2x+2)+3-2$
$=2(x-1)^2+1$

$S(1/+1)$

h) $y=-2x^2+4x$
$-2(x^2-2x+1)+2$
$=-2(x-1)^2+2$

$S(1/2)$

i) $y=2x^2+8x+9$
$=2(x^2+4x+4)-8+9$
$=2(x+2)^2+1$

$S(-2/1)$

k) $y=\frac{1}{2}x^2+x-\frac{1}{2}$
$=\frac{1}{2}(x^2+2x+1)-\frac{1}{2}-\frac{1}{2}=\frac{1}{2}(x+1)^2-1$

$S(-1/-1)$

l) $y=-\frac{1}{2}x^2+2x+3$
$=-\frac{1}{2}(x^2-4x+4)+3+2$
$=-\frac{1}{2}(x-2)^2+5$

$S(2/5)$

112/22 a) $S(2/1)$ oben b) $S(-2/+2)$ c) $S(2/-3)$

$y=(x-2)^2+1$
$y=x^2-4x+5$

$y=(x+2)^2+2$
$y=x^2+4x+6$

$y=x^2-4x+1$

d) $S(1/-3)$ unten e) $S(-3/-1)$ unten f) $S(3/-2)$

$y=-1(x-1)^2-3$
$y=-x^2+2x-4$

$y=-(x+3)^2-1$
$-x^2-6x-10$

$y=(x-3)^2-2$
$y=x^2-6x+7$

113/23 a) $y=a(x-2)^2+1$

wenn $x=0 \rightarrow y=4$ ~~a(4+1 4~~ $y=a(x-2)^2+1$
$4=4a+1$

$4a=3 \quad a=\frac{3}{4}$

$y=\frac{3}{4}(x-2)^2+1$
$=\frac{3}{4}(x^2-4x+4)+1$
$=\frac{3}{4}x^2-3x+\underbrace{3+1}_{4}$

b) $S(-3/2)$ $P(0/4)$

$y=a(x+3)^2+2$ wenn $x=0$: $y=9a+2=4$ $a=\frac{2}{9}$

$y=\frac{2}{9}x^2+1\frac{1}{3}x+4$

c) $S(4/-2)$ $P(0/0)$

$y=a(x-4)^2-2$
$0=16a-2$
$16a=2$
$a=\frac{1}{8}$

$y=\frac{1}{8}(x-4)^2-2$
$=\frac{1}{8}x^2-x+2-2$
$=\frac{1}{8}x^2-x$

Wo ist der Scheitelpunkt? (allgemein)

$y=ax^2+bx+c$
$=a\left(x^2+\frac{bx}{a}+\frac{b^2}{4a^2}\right)+c-\frac{b^2}{4a}$
$=a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$

$S\left(-\frac{b}{2a}\Big/-\frac{b^2-4ac}{4a}\right)$

22ghik, 23def

112/22 g) $S(4/5)$ unten h) $S(-4/+3)$ unten i) $S(-\frac{3}{2}/\frac{5}{2})$ unten

$y=-(x-4)^2+5$
$=-(x^2-8x+16)+5$
$=-x^2+8x-11$ ✓

$y=-(x+4)+3$
$=-(x^2+8x+16)+3$
$=-x^2-8x-13$ ✓

$y=-(x+\frac{3}{2})+\frac{5}{2}$
$=-(x^2+\frac{6}{2}x+\frac{9}{4})+\frac{5}{2}$
$=-x^2-3x+\frac{1}{4}$

k) $S(\frac{5}{2}/\frac{3}{2})$ unten

$y = -(x - \frac{5}{2})^2 + \frac{3}{2}$

$= -(x^2 - 5x + \frac{25}{4}) + \frac{3}{2}$

$= -x^2 + 5x - \frac{19}{4}$ ✓

113/23d) $y = a(x-2)^2 + 3$

$1 = a\,4 \;+3$

$1 = 4a + 3$

$-2 = 4a$

$a = -\frac{1}{2}$

$y = -\frac{1}{2}(x-2)^2 + 3$

$y = -\frac{1}{2}(x^2 - 4x + 4) + 3$

$y = -\frac{1}{2}x^2 + 2x - 2 + 3$

$\underline{\underline{y = -\frac{1}{2}x^2 + 2x + 1}}$

23e) $S(-1/-2)$ $P(0/0)$

$y = a(x+1)^2 - 2$

$0 = a - 2$

$a = 2$

$y = 2(x+1)^2 - 2$

$= 2(x^2 + 2x + 1) - 2$

$= 2x^2 + 4x + \cancel{2} - \cancel{2}$

$= \underline{\underline{2x^2 + 4x}}$

f) $S(-1/1)$ $P(1/0)$

$y = a(x+1)^2 + 1$

$0 = a(1+1)^2 + 1$

$0 = 4a + 1$

$a = -\frac{1}{4}$

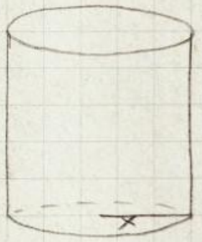
$y = -\frac{1}{4}(x+1)^2 + 1$

$= -\frac{1}{4}(x^2 + 2x + 1) + 1$

$= -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{1}{4} + 1$

$\underline{\underline{= -\frac{1}{4}x^2 - \frac{1}{2}x + \frac{3}{4}}}$

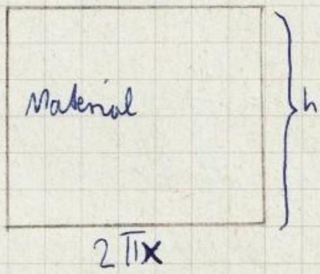
Volumen $= 300\pi$ cm³



$r = x$

$A = x^2\pi$

$h = \frac{300\pi}{x^2\pi} = \frac{300}{x^2}$



Materialverbrauch

$2x^2\pi + 2\cancel{x}\pi \cdot \frac{300}{x^{\cancel{2}}}$

Maxima – Minima

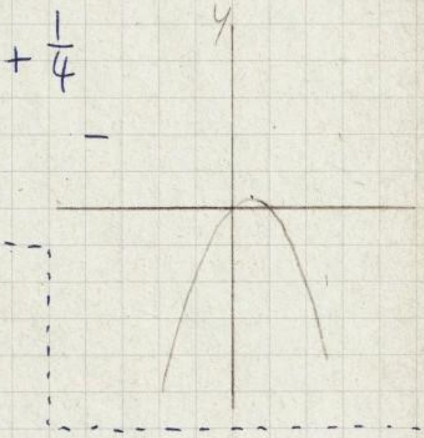
114/29) $f(x) = x - x^2$

$y = x - x^2$

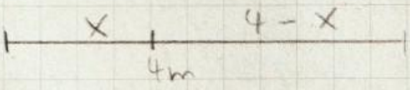
$= -x^2 + x$

$= -(x^2 - x + \frac{1}{4}) + \frac{1}{4}$ $\quad x = \frac{1}{2}$

$y = -(x - \frac{1}{2})^2 + \frac{1}{4}$



114/30)



$y = x^2 + (4-x)^2 =$ minimal

$= x^2 + 16 - 8x + x^2$

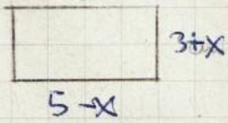
$= 2x^2 - 8x + 16$

$= 2(x^2 - 4x + 4) + 8 = 2(x-2)^2 + 8 \quad x = 2$

kleinste Wert: $S(2/8)$

in der Mitte der Strecke

114/31)



$(3+x)(5-x) =$ größte A $\qquad S(1/16)$

$15 + 2x - x^2$

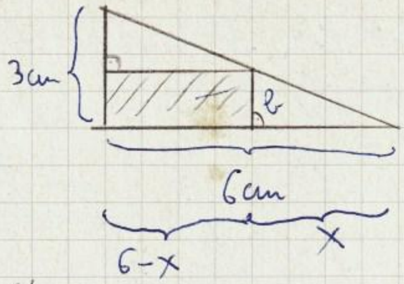
$= -x^2 + 2x + 15$

$= -(x^2 - 2x + 1) + 15 + 1$

$= -(x-1)^2 + 16$

$x = 1$

114/32)



$x : b = 6 : 3$

$6b = 3x$

$b = \frac{3x}{6} = \frac{x}{2}$

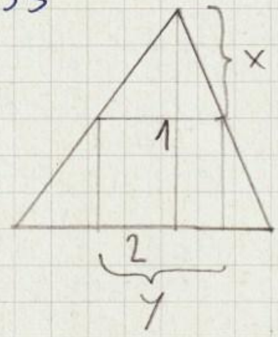
$A = (6-x)\cdot\frac{x}{2} = 3x - \frac{x^2}{2}$

$= -\frac{x^2}{2} + 3x = -\frac{1}{2}(x^2 - 6x + 9) + \frac{9}{2}$

$= -\frac{1}{2}(x-3)^2 + \frac{9}{2}$

$x = 3$

114/33 34



$A(y) = (1-x)y$

$x : y = 1 : 2$

$2x = y$

$(1-x)2x = 2x - 2x^2$

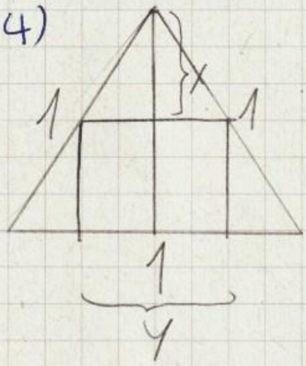
$= -2x^2 + 2x$

$= -2(x^2 - x + \frac{1}{4}) + \frac{1}{2}$

$= -2(x - \frac{1}{2})^2 + \frac{1}{2}$

$x = \frac{1}{2}$ $y = 1$

34)



$x : y = \frac{1}{\sqrt{3}} : 1$

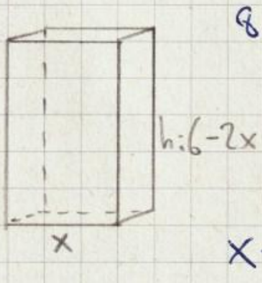
$x = \frac{1}{\sqrt{3}} - x$

$y = \sqrt{3}x$

$A(y) = \left(\frac{1}{\sqrt{3}} - x\right)\cdot y = \left(\frac{1}{\sqrt{3}} - x\right)\sqrt{3}x$

$= x - \sqrt{3}x^2 + \frac{1}{4}$

115/35a)



$8x + \underbrace{\frac{24-8x}{4}}_{6-2x} = 24$

$F = 2x^2 + 4x(6-2x)$ x = Grundkante

$= -6x^2 + 24x$

$y = -6(x^2 - 4x + 4) + 24$

$= -6(x-2)^2 + 24$

$x = 2$

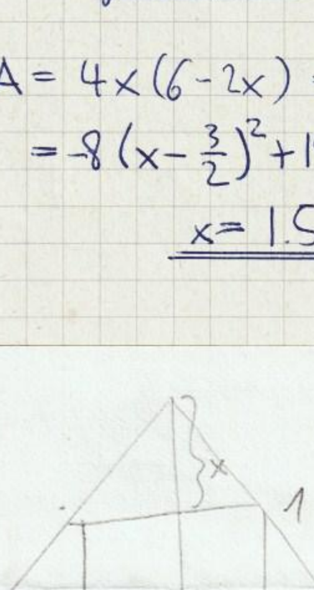
größte F: $24\ cm^2$

b) $A = 4x(6-2x) = -8x^2 + 24x = -8(x^2 - 3x + 2\frac{1}{4}) + 18$

$= -8(x - \frac{3}{2})^2 + 18$

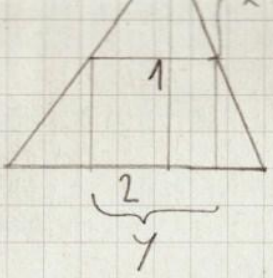
$x = 1.5$ $A = 18\ cm^2$

34)



$h = \frac{1}{2}\sqrt{3}$ $h : 1 = x : y = \frac{1}{2}\sqrt{3} : 1$

$\frac{1}{2}\sqrt{3}y = x$ $y = \frac{x}{\frac{1}{2}\sqrt{3}}$



$x : y = 1 : 2$

$2x = y$

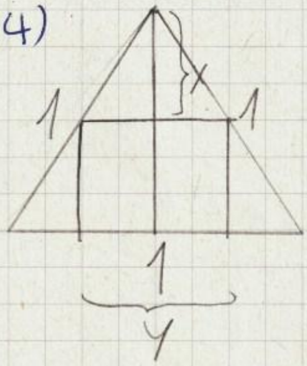
$\underline{\underline{x = \frac{1}{2}}}\ \underline{\underline{y = 1}}$

$-2x^2 + 2x$

$= -2(x^2 - x + \frac{1}{4}) + \frac{1}{2}$

$= -2(x - \frac{1}{2})^2 + \frac{1}{2}$

34)



$x : y = \frac{1}{\sqrt{3}} : 1$

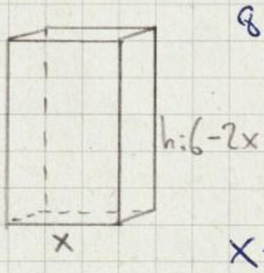
$x = \frac{1}{\sqrt{3}} - x$

$y = \sqrt{3}x$

$A(y) = (\frac{1}{\sqrt{3}} - x) \cdot y = (\frac{1}{\sqrt{3}} - x)\sqrt{3}x$

$= x - \sqrt{3}x^2 + \frac{1}{4}$

115/
35a)



$8x + \underbrace{\frac{24-8x}{4}}_{6-2x} = 24$

$F = 2x^2 + 4x(6-2x)$ x=Grundkante

$= -6x^2 + 24x$

$y = -6(x^2 - 4x + 4) + 24$

$= -6(x-2)^2 + 24$

$x = 2$

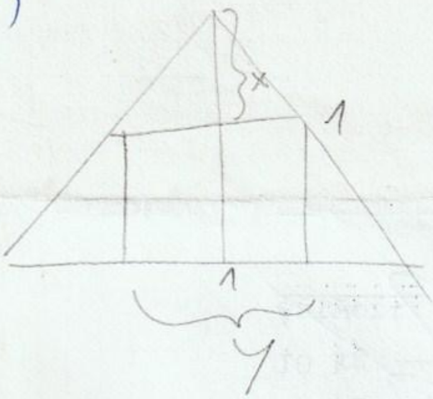
$\underline{\underline{\text{größte } F: 24\,cm^2}}$

b) $A = 4x(6-2x) = -8x^2 + 24x = -8(x^2 - 3x + 2\frac{1}{4}) + 18$

$= -8(x - \frac{3}{2})^2 + 18$

$\underline{\underline{x = 1.5}}$ $\underline{\underline{A = 18\,cm^2}}$

34)



$h = \frac{1}{2}\sqrt{3}$ $h : 1 = x : y = \frac{1}{2}\sqrt{3} : 1$

$\frac{1}{2}\sqrt{3}\,y = x$ $y = \frac{x}{\frac{1}{2}\sqrt{3}}$

$(\frac{1}{2}\sqrt{3} - x)\frac{x}{\frac{1}{2}\sqrt{3}} = x - \frac{x^2}{\frac{1}{2}\sqrt{3}} = -\frac{x^2}{\frac{1}{2}\sqrt{3}} + x$

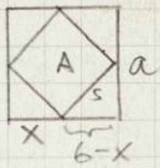
$= -\frac{1}{\frac{1}{2}\sqrt{3}}(x^2 - x + \frac{1}{4}) + \frac{1}{2\sqrt{3}}$

$= -\frac{1}{\frac{1}{2}\sqrt{3}}(x - \frac{1}{2})^2 + \frac{1}{2\sqrt{3}}$ $\underline{\underline{x = \frac{1}{2}}}$

$\frac{1}{2}\sqrt{3} : 1 = \frac{1}{2} : y$

$\underline{\underline{y = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}}}} = \underline{\underline{\frac{1}{\sqrt{3}}}}$

115/36)



$a = 6\,cm$

$A = (6-x)^2 + x^2 = 36 - 12x + 2x^2$

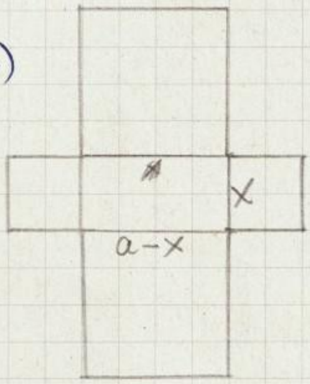
$s = \sqrt{(6-x)^2 + x^2}$

$y = 2(x^2 - 6x + 9) + 18$

$A = 18\,cm^2 = 2(x-3)^2 + 18$

$x = 3$

115/37)



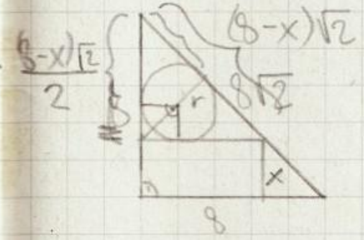
$U = 2a$

$$F(x) = 2x^2 + 2(a-x)^2$$
$$= 2[x^2 + (a-x)^2]$$
$$= 2[x^2 + a^2 - 2ax + x^2]$$
$$= 2(2x^2 - 2ax + a^2)$$
$$= 4(x^2 - ax + \tfrac{1}{4}a^2) - a^2$$
$$= 4\left(x - \frac{a}{2}\right)^2$$

$x = \frac{a}{2}$

$\Rightarrow$ Quadrat

Gegeben ist ein rechtwinkl. gleichsch. Δ



Rechteck: $l : 8 = (8-x) : 8 \quad l = 8 - x$

$$r = (8-x) - \frac{(8-x)\sqrt{2}}{2} = (8-x)\left[1 - \frac{\sqrt{2}}{2}\right] = (8-x)\left(\frac{2-\sqrt{2}}{2}\right)$$

$$F = 8x - x^2 + (8-x)^2\left(\frac{2-\sqrt{2}}{2}\right)^2 \cdot \pi = 8x - x^2 + \pi\left[(64 - 16x + x^2)\left(\frac{6-4\sqrt{2}}{4}\right)\right]$$

$$= 8x - x^2 + \pi\left[16(6-4\sqrt{2}) + x^2\left(\frac{6-4\sqrt{2}}{4}\right) - 4x(6-4\sqrt{2})\right]$$

$$= -x^2 + x^2 \cdot \frac{6-4\sqrt{2}}{4}\pi + 8x - 4x(6-4\sqrt{2})\pi + 16\pi(6-4\sqrt{2})$$

$$= x^2\Big[\underbrace{-1 + \frac{(6-4\sqrt{2})\pi}{4}}_{-0.730493395}\Big] + x\Big[\overbrace{8 - 24\pi + 16\sqrt{2}\pi}^{3.?68793033}\Big] + \overbrace{16\pi(6-4\sqrt{2})}^{17.248387}$$

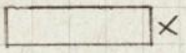
$$= x^2\left[\frac{-4 + 6\pi - 4\sqrt{2}\pi}{4}\right] + x\left[8 - 24\pi + 16\sqrt{2}\pi\right] + 16\pi(6-4\sqrt{2})$$

$$= (x^2 + x)[-1 + 1.5\pi - \sqrt{2}\pi + 8 - 24\pi + 16\sqrt{2}\pi] + 96\pi - 64\sqrt{2}\pi$$

$$= (x^2 + x)\cancel{[7 + 73.5\pi - 49\sqrt{2}\pi]}(7 - 22.5\pi + 15\sqrt{2}\pi) + 96\pi - 64\sqrt{2}\pi$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - ac}}{2a}$$ x 27.524 y 21.3

115/38)

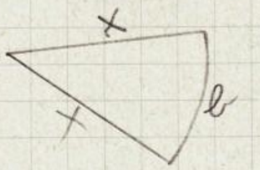


$$\frac{4-2x}{2} = 2-x$$

$$y = x(2-x) = 2x - x^2 = -x^2 + 2x = -(x^2-2x+1)+1$$
$$= -(x-1)^2 + 1$$

$$\underline{\underline{x = 1}}$$

115/40)



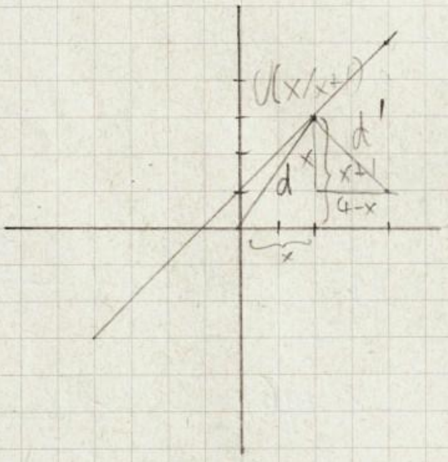
$$F = \frac{b \cdot r}{2} \qquad b = 2-2x \qquad F = \frac{2(1-x)x}{\cancel{2}} = x(1-x)$$
$$= x - x^2 = -x^2 + x$$
$$= -(x^2 - x + \tfrac{1}{4}) + \tfrac{1}{4} = -(x-\tfrac{1}{2})^2 + \tfrac{1}{4}$$

$$\underline{\underline{x = \frac{1}{2}}}$$

115/41)
$$s = vt - \frac{g}{2}t^2 = -\frac{g}{2}\left(t^2 - \frac{2vt}{g} + \frac{v^2}{g^2}\right) + \frac{v^2}{2g}$$
$$-\frac{g}{2} \cdot \frac{v^2}{g^2}$$
$$= -\frac{g}{2}\left(t - \frac{v}{g}\right)^2 + \frac{v^2}{2g}$$

<u>höchster Punkt nach $\frac{v}{g}$ Sekunden</u>

115/42)



$$x - y + 1 = 0$$
$$y = x + 1$$

$$d = \sqrt{x^2 + (x+1)^2} = \sqrt{(x^2 + x^2 + 2x + 1)}$$
$$f(x) = 2x^2 + 2x + 1$$
$$= 2(x^2 + x + \tfrac{1}{4}) + \tfrac{1}{2}$$
$$= 2(x + \tfrac{1}{2})^2 + \tfrac{1}{2} \qquad \underline{\underline{x = -\tfrac{1}{2}}}$$

$$d' = \sqrt{x^2 + (4-x)^2} = \sqrt{2x^2 + 16 - 8x} \qquad x = 2, \; y = 3$$
$$f(x) = 2(x^2 - 4x + 4) + 8 = 2(x-2)^2 + 8$$

116/43)
$$y = (m+1)x^2 - 4mx + m + 6$$
$$y = (m+1)\left(x^2 - \frac{4mx}{m+1} + \frac{4m^2}{(m+1)^2}\right) + m + 6 - \frac{4m^2}{m+1} = (m+1)\left(x - \frac{2m}{m+1}\right)^2 + m + 6 - \frac{4m^2}{m+1}$$

$$5 = m + 6 - \frac{4m^2}{m+1} \qquad 0 = 3m^2 - 2m - 1$$
$$0 = m + 1 - \frac{4m^2}{m+1}$$
$$0 = m^2 + 2m + 1 - 4m^2$$

$$m_{1/2} = \frac{2 \pm \sqrt{4+12}}{6} = \frac{2 \pm 4}{6} = \begin{cases} 1 \\ -\frac{1}{3} \end{cases}$$

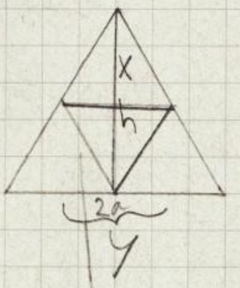
Resultate in die 1. Gleichung eingesetzt:

$$y = 2x^2 - 4x + 7 = 2(x^2 - 2x + 1) + 5 = 2(x-1)^2 + 5$$

$$y = \frac{2}{3}x^2 + \frac{4}{3}x + 5\frac{2}{3} = \frac{2}{3}(x^2 + 2x + 1) + 5 = \frac{2}{3}(x+1)^2 + 5$$

44, 39

115/39)



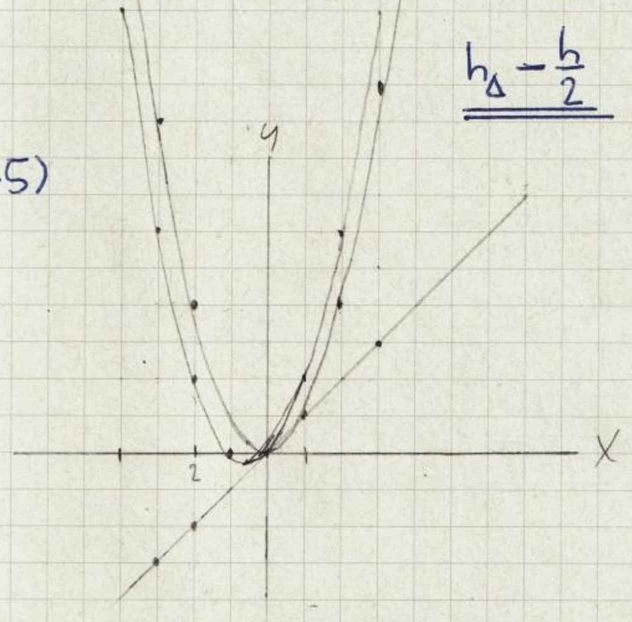
$$(h-x)\frac{y}{2} = (h-x)\frac{x\sqrt{3}}{2} = \frac{\sqrt{3}hx}{2} - \frac{\sqrt{3}x^2}{2}$$

$$= -\frac{\sqrt{3}x^2}{2} + \frac{\sqrt{3}hx}{2} = \frac{1}{2}\sqrt{3}(x^2 - hx + \frac{1}{4}h^2) + \frac{1}{8}\sqrt{3}h$$

$$= \frac{1}{2}\sqrt{3}\left(x - \frac{h}{2}\right)^2$$

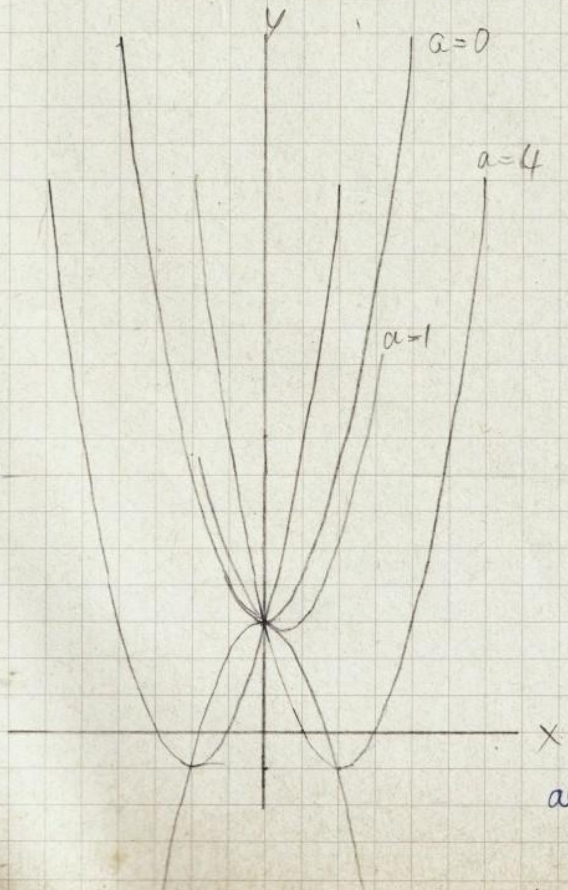
$$\underline{\underline{h_\Delta - \frac{h}{2}}}$$

116/45)



$$y = \left(x^2 + x + \frac{1}{4}\right) \cancel{+1} - \frac{1}{4} = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$

116/48)



$$y = x^2 + 4x + 3$$
$$y = (x+2)^2 - 1$$

Alle Scheitelpunkte liegen auch auf einer Normalparabel.

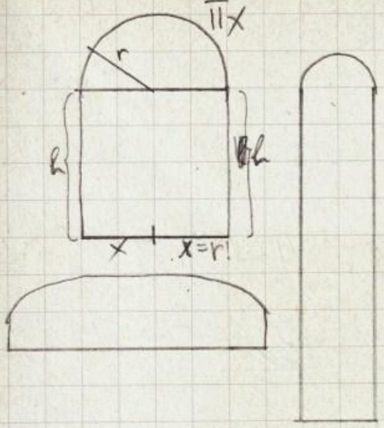
$$y = x^2 + ax + 3 = \left(x^2 + ax + \frac{a^2}{4}\right) + 3 - \frac{a^2}{4} = \left(x + \frac{a}{2}\right)^2 + 3 - \frac{a^2}{4}$$

wenn $x_s = -\frac{a}{2}$

dann $y_s = 3 - \frac{a^2}{4}$

$$y_s = 3 - x_s^2$$

allgemein: $y = b - x^2 = -x^2 + b$



Bei welcher Form niedrigste Materialkosten für Ausbetonierung?

Gegeben ist der Umfang U. Bei welchen Maßen ist der Querschnitt maximal?

$$U - x(2+\pi) \qquad h = \frac{U - x(2+\pi)}{2}$$

$$F = 2xh + \frac{x^2\pi}{2} = Ux - x^2(2+\pi) + \frac{\pi x^2}{2} = -x^2\left[2+\frac{\pi}{2}\right] + Ux$$

$$= -\left(2+\frac{\pi}{2}\right)\left[x^2 - x\frac{U}{2+\frac{\pi}{2}}\right] = -\left(2+\frac{\pi}{2}\right)\left[x^2 - x\frac{2U}{4+\pi}\right] + \left(2+\frac{\pi}{2}\right)\left(\frac{U^2}{(4+\pi)^2}\right)$$

$$= -\left(2+\frac{\pi}{2}\right)\left[x^2 - x\frac{2U}{4+\pi}\right] + \frac{1}{2}(4+\pi)\left(\frac{U^2}{(4+\pi)^2}\right)$$

$$= -\left(2+\frac{\pi}{2}\right)\left[x - \frac{U}{4+\pi}\right]^2 + \frac{U^2}{2(4+\pi)}$$

$$\underline{\underline{x = \frac{U}{4+\pi}}} \qquad h = \frac{U - \frac{U}{4+\pi}(2+\pi)}{2} = \frac{4U + \pi U - 2U - U\pi}{2(4+\pi)}$$

$$\underline{\underline{h = \frac{U}{4+\pi}}}$$

2