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Nonnenweg 7  
4055 Basel

Mit  
Conto

Rechen-Algebra GaMNb







Gleichungssysteme zweiten Grades

Zwei Gleichungen zweiten Grades mit zwei Unbekannten bilden ein Gleichungssystem zweiten Grades.

$$\begin{cases} a_1x^2 + b_1xy + c_1y^2 + d_1x + e_1y + f_1 = 0 \\ a_2x^2 + b_2xy + c_2y^2 + d_2x + e_2y + f_2 = 0 \end{cases}$$

Ist eine Gleichung linear, so ist die Lösung einfach:

Bsp.: 117/24)  $\begin{cases} 3x + y + 9 = 0 \\ x^2 + 2y^2 - x + 3y - 15 = 0 \end{cases} \quad y = -3x - 9 \quad \begin{matrix} y_1 = -3 \\ y_2 = \frac{9}{19} \end{matrix}$

$$\begin{aligned} x^2 + 2(-3x - 9)^2 - x + 3(-3x - 9) - 15 &= 0 \\ x^2 + 2(9x^2 - 54x + 81) - x + 3(-3x - 9) - 15 &= 0 \\ x^2 + 18x^2 - 108x + 162 - x - 9x - 27 - 15 &= 0 \end{aligned}$$

$$19x^2 + 98x + 120 = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-98 \pm 22}{38} = \begin{cases} -2 \\ -3\frac{3}{19} \end{cases}$$

$$L = \left\{ (-2 | -3); \left(-3\frac{3}{19} \mid \frac{9}{19}\right) \right\}$$

Sind beide Gleichungen quadratisch, so kann man oft eine lineare Gleichung herbeiführen.

Bsp.: 118/46)  $\begin{cases} x^2 - xy + y^2 - 39 = 0 \\ 2x^2 - 3xy + 2y^2 - 43 = 0 \end{cases} \quad \begin{aligned} & \cdot 2 + x^2 - x\frac{35}{x} + \left(\frac{35}{x}\right)^2 - 39 = 0 \\ & - x^2 - 35 + \frac{1225}{x^2} - 39 = 0 \quad | \cdot x^2 \end{aligned}$

$$\begin{aligned} 2x^2 - 2xy + 2y^2 - 78 &= 0 \quad + \\ 2x^2 - 3xy + 2y^2 - 43 &= 0 \quad - \end{aligned}$$

$$xy - 35 = 0$$

$$y = \frac{35}{x}$$

$$x^4 - 35x^2 + 1225 - 39x^2 = 0$$

$$x^4 - 74x^2 + 1125 = 0 \quad x^2 = u$$

$$u^2 - 74u + 1125 = 0$$

$$u_{1/2} = \frac{74 \pm 24}{2} = \begin{cases} 49 \\ 25 \end{cases}$$

$$x^2 = 49 \Rightarrow \begin{matrix} x_1 = 7 & y_1 = 5 \\ x_2 = -7 & y_2 = -5 \end{matrix}$$

$$x^2 = 25 \Rightarrow \begin{matrix} x_3 = 5 & y_3 = 7 \\ x_4 = -5 & y_4 = -7 \end{matrix}$$

$$L = \left\{ (7/5); (-7/-5); (5/7); (-5/-7) \right\}$$



$$117/7) \begin{cases} y-x=8 \\ xy+15=0 \end{cases} \quad y=8+x$$

$$\begin{aligned} x(8+x) &= -15 \\ 8x+x^2 &= -15 \\ x^2+8x+16-16 &= -15 \\ x^2+8x &+16-16 &= -15 \\ x^2+8x+15 &= 0 \end{aligned}$$

$$x_{1/2} = \frac{-8 \pm 2}{2} \begin{cases} -3 & y_1=5 \\ -5 & y_2=3 \end{cases}$$

$$L = \{(-3/5), (-5/3)\} \quad \checkmark$$

$$117/10) \begin{cases} x^2+y^2=58 \\ x+y=10 \end{cases} \quad x=10-y \quad y_{1/2} = \frac{20 \pm 8}{4} \begin{cases} 7 \\ 3 \end{cases}$$

$$(10-y)^2 + y^2 = 58$$

$$100 - 20y + y^2 + y^2 = 58$$

$$2y^2 - 20y + 42 = 0$$

$$\begin{aligned} x_1 &= 3 \\ x_2 &= 7 \end{aligned}$$

$$L = \{(3/7), (7/3)\} \quad \checkmark$$

$$117/22) \begin{cases} x-2y=4 \\ x^2+y^2+x-y-8=0 \end{cases} \quad x=4+2y$$

$$\begin{aligned} (4+2y)^2 + y^2 + (4+2y) - y - 8 &= 0 \\ 16 + 16y + 4y^2 + y^2 + 4 + 2y - y - 8 &= 0 \end{aligned}$$

$$5y^2 + 17y + 12 = 0$$

$$y_{1/2} = \frac{-17 \pm 7}{10} = \begin{cases} -1 \\ -2.4 \end{cases}$$

$$\begin{aligned} x_1 &= 2 \\ x_2 &= -0.8 \end{aligned}$$

$$L = \{(2/-1), (-0.8/-2.4)\} \quad \checkmark$$

$$118/38) \begin{cases} 2x^2 - 3y^2 = 6 \\ 3x^2 - 2y^2 = 19 \end{cases} \begin{aligned} \cdot 3 & \quad 2x^2 - 12 = 6 \\ \cdot 2 & \quad 2x^2 = 18 \\ & \quad x^2 = 9 \\ & \quad x_1 = 3 \\ & \quad x_2 = -3 \end{aligned}$$

$$6x^2 - 9y^2 = 18 \quad +$$

$$6x^2 - 4y^2 = 38 \quad - \quad L = \{(3/2), (-3/2), (-3/-2), (3/-2)\}$$

$$\begin{aligned} -y^2 &= 20 \\ y^2 &= 4 \\ y_1 &= 2 \\ y_2 &= -2 \end{aligned}$$

$$118/41) \begin{cases} x^2 - a^2y^2 = b \\ bx^2 + ay^2 = 1 \end{cases} \cdot b$$

$$bx^2 + a \frac{1-b^2}{a^2b+a} = 1$$

$$bx^2 + \frac{1-b^2}{ab+1} = 1$$

$$\begin{cases} bx^2 - a^2by^2 = b^2 \\ bx^2 + ay^2 = 1 \end{cases} \begin{aligned} + & \quad \frac{ab^2x + b^2x^2 + 1 - b^2}{ab+1} = 1 \\ - & \quad bx^2 = 1 - \frac{1-b^2}{ab+1} \end{aligned}$$

$$\frac{ab^2x + b^2x^2 + 1 - b^2}{ab+1} = 1$$

$$bx^2 = 1 - \frac{1-b^2}{ab+1}$$

$$-a^2by^2 - ay^2 = b^2 - 1$$

$$a^2by^2 + ay^2 = 1 - b^2$$

$$y^2(a^2b+a) = 1 - b^2$$

$$bx^2 = \frac{ab+1 - 1 + b^2}{ab+1}$$

$$- \frac{ab+b^2}{ab+1}$$

$$y^2 = \frac{1-b^2}{a^2b+a}$$

$$x = \pm \sqrt{\frac{a+b^2}{ab+1}}$$

$$y = \pm \sqrt{\frac{1-b^2}{a^2b+a}} = \frac{(1+b)(1-b)}{a(1+b)}$$



$$118/46) \quad x^2 - xy + y^2 - 39 = 0 \quad | \cdot 2$$

$$2x^2 - 3xy + 2y^2 - 43 = 0$$

$$\begin{array}{r} 2x^2 - 2xy + 2y^2 - 78 = 0 \quad | + \\ 2x^2 - 3xy + 2y^2 - 43 = 0 \quad | - \end{array}$$

$$xy - 35 = 0 \quad \frac{35}{x} = y$$

$$\begin{array}{l|l} y_1 = 7 & x_1 = 5 \\ y_2 = 5 & x_2 = 7 \\ y_3 = -7 & x_3 = -5 \\ y_4 = -5 & x_4 = -7 \end{array}$$

$$\frac{35^2}{y^2} - 35 + y^2 - 39 = 0 \quad | \cdot y^2$$

$$35^2 - 35y^2 + y^4 - 39y^2 = 0$$

$$y^4 - 74y^2 + 35^2 = 0 \quad y^2 = u$$

$$u^2 - 74u + 35^2 = 0$$

$$u_{1/2} = \frac{74 \pm \sqrt{5476 - 4900}}{2} \quad \begin{array}{l} 49 \\ 25 \end{array}$$

$$119/62a) \quad x^2 + y^2 = 425$$

$$\log x + \log y = 2$$

$$x^2 + y^2 = 425$$

$$\log xy = 2$$

$$x^2 + y^2 = 425$$

$$xy = 100$$

$$x = \frac{100}{y} \quad u_{1/2} = \frac{425 \pm \sqrt{425^2 - 40'000}}{2} = \frac{425 \pm 375}{2} = \begin{array}{l} 400 \\ 25 \end{array}$$

$$\frac{100^2}{y^2} + y^2 = 425$$

$$y^4 - 425y^2 + 10'000 = 0$$

$$u^2 - 425u + 10'000 = 0$$

$$\begin{array}{l|l} y_1 = 20 & x_1 = 5 \\ y_2 = -20 & x_2 = -5 \\ y_3 = 5 & x_3 = 20 \\ y_4 = -5 & x_4 = -20 \end{array}$$

$$124/110) \quad \begin{array}{l} x + y + \sqrt{x+y} = 30 \\ \sqrt{xy} = 12 \end{array} \quad \begin{array}{l} x + y = u^2 \\ xy = 144 \end{array}$$

$$u^2 + u - 30 = 0$$

$$(u+6)(u-5) = 0 \rightarrow u_1 = -6 \quad u_2 = 5$$

$$x + y = 36 \quad | \quad x + y = 25$$

$$xy = 144 \quad | \quad xy = 144$$

$$x = \frac{144}{y} \quad \frac{144}{y} + y = 36$$

$$y^2 - 36y + 144 = 0$$

$$y_{1/2} = \frac{36 \pm \sqrt{720}}{2} = \frac{36 \pm 12\sqrt{5}}{2} = 18 \pm 6\sqrt{5}$$

$$x_{1/2} = 18 \mp 6\sqrt{5}$$

$$\begin{array}{l} x + y = 25 \\ xy = 144 \end{array} \quad x = \frac{144}{y}$$

$$\frac{144}{y} + y = 25 \quad | \cdot y$$

$$y^2 - 25y + 144 = 0$$

$$y_{1/2} = \frac{25 \pm \sqrt{625 - 576}}{2} \quad \begin{array}{l} 16 \\ 9 \end{array}$$

$$\begin{array}{l} x_1 = 9 \\ x_2 = 16 \end{array}$$



$$124/111) \begin{cases} 2x^2 - 3y^2 - 5xy = 0 \\ 3x^2 - 8xy - 3 = 0 \end{cases} \Rightarrow (2x+y)(x-3y) = 0$$

$$3x^2 - 8xy - 3 = 0$$

Entweder  $2x+y=0$  od.  $x-3y=0$

$$3x^2 - 8xy - 3 = 0 \quad 3x^2 - 8xy - 3 = 0$$

$$y = -2x$$

$$3x^2 + 16x^2 - 3 = 0$$

$$19x^2 - 3 = 0$$

$$x^2 = \frac{3}{19}$$

$$\underline{x_{1/2} = \pm \sqrt{\frac{3}{19}} \quad y_{1/2} = \mp 2\sqrt{\frac{3}{19}}}$$

$$x = 3y$$

$$27y^2 - 24y^2 - 3 = 0$$

$$3y^2 = 3$$

$$\underline{y_{3/4} = \pm 1 \quad x_{3/4} = \pm 3}$$

andere Lösung:

Man betrachte  $y$  als gegeben.

$$\rightarrow x_{1/2} = \frac{5y \pm \sqrt{25y^2 + 24y^2}}{4} = \frac{5y \pm 7y}{4} = \begin{cases} 3y \\ -\frac{1}{2}y \end{cases}$$

$$124/113) \begin{cases} 5x^2 - 6xy - 8y^2 = 0 \\ x^2 - 3y^2 = 1 \end{cases}$$

$$x = 2y$$

$$4y^2 - 3y^2 = 1$$

$$y^2 = 1$$

$$y = \pm 1$$

$$x_{1/2} = \pm 2$$

$$x_{3/4} = \mp \frac{4}{5} i \sqrt{\frac{25}{59}}$$

$$x_{1/2} = \frac{6y \pm \sqrt{36y^2 + 160y^2}}{10} = \frac{6y \pm 14y}{10} = \begin{cases} 2y \\ -\frac{4}{5}y \end{cases}$$

$$x = -\frac{4}{5}y$$

$$\frac{16}{25}y^2 - 3y^2 = 1$$

$$-2\frac{9}{25}y^2 = 1$$

$$y^2 = \pm \frac{25}{59}$$

$$y_{3/4} = \pm i \sqrt{\frac{25}{59}}$$



$$124/114) \begin{cases} (x-3)^2 + (y-3)^2 = 25 \\ x + y = 13 \end{cases} \quad x = 13 - y$$

$$(10-y)^2 + (y-3)^2 = 25$$

$$100 - 20y + y^2 + y^2 - 6y + 9 = 25$$

$$2y^2 - 26y + 84 = 0$$

$$(y^2 - 13y) + 42 = 0$$

$$(y-6)(y-7) = 0$$

$$y_1 = 6, y_2 = 7$$

$$x_1 = 7, x_2 = 6$$

$$124/117) \begin{cases} x^2 + y^2 + x - y = 114 \\ (x^2 + y^2)(x - y) = 113 \end{cases}$$

$$x^2 + y^2 = u$$

$$x - y = v$$

$$\begin{cases} u + v = 114 \\ uv = 113 \end{cases} \Rightarrow u = 114 - v$$

$$(114 - v)v = 113$$

$$-v^2 + 114v - 113 = 0 \quad v_{1/2} = \frac{-114 \pm \sqrt{114^2 - 4 \cdot 113}}{-2} = \frac{-114 \pm 112}{-2} \begin{matrix} 1 \\ 113 \end{matrix}$$

$$u_1 = 114 - 113 = 1 \quad v_1 = 113$$

$$u_2 = 114 - 1 = 113 \quad v_2 = 1 \quad x - y = 1$$

$$\begin{cases} x^2 + y^2 = 113 \\ xy = 1 \end{cases} \quad (x^2 + y^2 + 1 = 114) \quad \uparrow v=1$$

$$(1+y)^2 + y^2 = 113 \quad (y-7)(y+8) = 0$$

$$2y^2 + 2y - 112 = 0 \quad y_1 = 7 \quad x_1 = 8$$

$$y^2 + y - 56 = 0 \quad y_2 = -8 \quad x_2 = -7$$

$$y_{1/2} = \frac{-113 \pm \sqrt{113^2 - 4 \cdot 6384}}{2}$$



112 115 116

124/112

$$\begin{cases} x^2 + 3xy = 0 \\ x^2 - xy + y^2 = 3 \end{cases}$$

$$x(x+3y) = 0$$

$$x_1 = 0 \text{ od. } x+3y=0$$

$$x = -3y$$

$$0 - 0 + y^2 = 3$$

$$\begin{aligned} y &= \sqrt{3} \\ y_1 &= \sqrt{3} \\ y_2 &= -\sqrt{3} \end{aligned}$$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \end{aligned}$$

$$x_3 = -3\sqrt{\frac{3}{13}}$$

$$9y^2 + 3y^2 + y^2 = 3$$

$$13y^2 = 3$$

$$y_{3+4} = \pm \sqrt{\frac{3}{13}}$$

$$y_3 = \sqrt{\frac{3}{13}}$$

$$x_3 = -3\sqrt{\frac{3}{13}}$$

$$y_4 = -\sqrt{\frac{3}{13}}$$

$$x_4 = 3\sqrt{\frac{3}{13}}$$

124/115

$$(x-5)^2 + (y+5)^2 = 74$$

$$x - y = 8$$

$$x = 8 + y$$

$$\begin{aligned} x_1 &= 10 \\ x_2 &= -2 \end{aligned}$$

$$(8+y-5)^2 + (y+5)^2 = 74$$

$$(3+y)^2 + (y+5)^2 = 74$$

$$9 + 6y + y^2 + y^2 + 10y + 25 = 74$$

$$34 + 16y + 2y^2 = 74$$

$$2y^2 + 16y - 40 = 0$$

$$y_{1/2} = \frac{-16 \pm \sqrt{256 + 320}}{4} = \frac{-16 \pm 24}{4} \begin{cases} 2 \\ -10 \end{cases}$$

124/116

$$x^2 + y^2 - 6x + 2y = 27$$

$$x = \frac{4y}{5}$$

$$\underline{x_1 = 4} \quad \underline{x_2 = -2.634 + 1.63i} = -2 \frac{26}{41}$$

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{16y^2}{25} + y^2 - \frac{24y}{5} + 2y = 27 + 25$$

$$16y^2 + 25y^2 - 120y + 50y = 675$$

$$41y^2 - 70y - 675 = 0$$

$$y_{1/2} = \frac{70 \pm \sqrt{4900 + 110700}}{82} = \frac{70 \pm 340}{82}$$

$$= \begin{cases} 5 \\ -3 \frac{13}{41} \end{cases}$$

124/119

$$3x^2 - 3xy + 6y^2 + y = 51 \quad \cdot 2$$

$$12x^2 - 2xy + 4y^2 - x = 30 \quad \cdot 3$$

$$6x^2 - 6xy + 12y^2 + 2y = 102 \quad +$$

$$6x^2 - 6xy + 12y^2 - 3x = 90 \quad -$$

$$3x + 2y = 12$$

$$y = \frac{12 - 3x}{2}$$

$$3x^2 - 3x \frac{12-3x}{2} + 6 \left( \frac{12-3x}{2} \right)^2 + \frac{12-3x}{2} = 51$$

$$3x^2 - 3x \frac{12-3x}{2} + 6 \frac{144 - 72x + 9x^2}{4} + \frac{12-3x}{2} = 51 \quad \cdot 4$$

$$12x^2 - 12x(12-3x) + 6(144 - 72x + 9x^2) + 2(12-3x) = 204$$

$$12x^2 - 144x + 36x^2 + 864 - 432x + 54x^2 + 24 - 6x = 204$$

$$12x^2 - 288x + 72x^2 + 864 - 432x + 54x^2 + 24 - 6x = 204$$

$$-6x^2 - 726x + 684 = 0 \quad \cdot 1:2$$

$$6x^2 + 121x - 114 = 0$$



$$3x^2 - 3x \frac{12-3x}{2} + 6 \frac{144-72x+9x^2}{4} + \frac{12-3x}{2} = 511 \cdot 4$$

$$12x^2 - 6x(12-3x) + 6(144-72x+9x^2) + 2(12-3x) = 204$$

$$12x^2 - 72x + 18x^2 + 864 - 432x + 54x^2 + 24 - 6x = 204$$

$$84x^2 - 510x + 888 - 204 = 0$$

$$x_{1,2} = \frac{+510 \pm \sqrt{260100 - 291648 - 229'824}}{168} = \frac{510 \pm 174}{168} = \begin{cases} 4 \frac{1}{14} \\ 2 \frac{7}{28} \end{cases}$$

$$y_1 = \frac{2 \frac{13}{28} - \frac{3}{28}}{3} = \frac{10}{28} = \frac{5}{14}$$

$$y_2 = 3$$

$$x_1 = \frac{114}{28} = \frac{27}{7}$$

$$x_2 = \frac{56}{28} = 2$$

$$y_1 = \frac{168}{14} = \frac{171}{14} = \frac{-3}{28}$$

$$124 \quad 120 \quad \left. \begin{array}{l} 4x^2 - 3y = 1 \\ 4y^2 + 3x = 1 \end{array} \right| \begin{array}{l} = \frac{4x^2-1}{3} \\ = \frac{4y^2-1}{3} \end{array}$$

$$4 \left( \frac{4x^2-1}{3} \right)^2 + 3x = 1$$

$$4 \frac{64x^4 - 32x^2 + 4}{9} + 3x = 1 \quad | \cdot 9$$

$$64x^4 - 32x^2 + 4 + 27x = 9$$

$$64x^4 - 32x^2 + 27x - 5 = 0$$

andere Lösung?

noch andere Lösungen gehen nicht, wenn man die Gleichung dividiert.

$$(64x^4 - 32x^2 + 27x - 5) : (x+1) = 64x^3 - 64x^2 + 32x - 5$$

$$\begin{array}{r} 64x^4 + 64x^3 \\ -64x^3 - 32x^2 + 27x - 5 \\ +64x^3 + 64x^2 \\ \hline 0 + 32x^2 + 27x - 5 \\ -32x^2 - 32x \\ \hline 0 - 5x - 5 \\ +5x + 5 \\ \hline 0 \end{array}$$

Faktorisieren:  $(x+1)(64x^3 - 64x^2 + 32x - 5) = 0$

$$x^3 - x^2 + \frac{1}{2}x - \frac{5}{64} = 0 \quad x = \frac{1}{4}$$

$$\frac{1}{64} - \frac{1}{16} + \frac{1}{8} - \frac{5}{64} = 0$$

Division durch  $(x - \frac{1}{4})$  od. die obere durch

$$(4x-1)$$

$$x = \frac{204}{-204} \quad (64x^3 - 64x^2 + 32x - 5) : (4x-1) = 16x^2 - 12x + 5$$

$$\begin{array}{r} 64x^3 - 64x^2 + 32x - 5 \\ -64x^3 + 16x^2 \\ \hline -48x^2 + 32x - 5 \\ 48x^2 - 12x \\ \hline 20x - 5 \\ 0 \end{array}$$

Faktorisierung

$$(x+1)(4x-1)(16x^2 - 12x + 5) = 0$$

$$y = \frac{4x^2-1}{3}$$

$$y_1 = 1$$

$$y_2 = -\frac{1}{4}$$

$$y_3 = \frac{-3 + i\sqrt{11}}{8}$$

$$y_4 = \frac{-3 - i\sqrt{11}}{8}$$

$$16x^2 - 12x + 5 = 0$$

$$x_{3/4} = \frac{12 \pm \sqrt{144 - 320}}{32} = \frac{12 \pm \sqrt{-176}}{32} = \frac{12 \pm 4i\sqrt{11}}{32} = \frac{3 \pm i\sqrt{11}}{8} = \frac{3}{8} \pm \frac{i\sqrt{11}}{8}$$



$$3x^2 - 3x + 6 \frac{12-3x}{2} + 6 \frac{14-7x+9x^2}{4} + 2 \frac{12-3x}{2} = 511 \cdot 4$$

$$12x^2 - 6x(12-3x) + 6(14-7x+9x^2) + 2(12-3x) = 204$$

$$12x^2 - 72x + 18x^2 + 84 - 42x + 54x^2 + 24 - 6x = 204$$

$$84x^2 - 510x + 868 - 204 = 0$$

$$x_{1/2} = \frac{+510 \pm \sqrt{260100 - 291648 - 229924}}{168} = \frac{510 \pm 174}{168} = \begin{cases} 4 \frac{1}{4} \\ 2 \frac{7}{8} \end{cases}$$

$$y_1 = \frac{2 \frac{13}{28} - \frac{3}{28}}{3} = \frac{10}{28} = \frac{5}{14}$$

$$y_2 = 3$$

$$x_1 = \frac{114}{28} = \frac{27}{7}$$

$$x_2 = \frac{56}{28} = 2$$

$$y_1 = \frac{168}{14} = \frac{171}{14} = \frac{-3}{28}$$

$$124 \quad 120 \quad \left| \begin{array}{l} 4x^2 - 3y = 1 \\ 4y^2 + 3x = 1 \end{array} \right| = \frac{4x^2 - 1}{3}$$

$$4y^2 + 3x = 1$$

$$4 \left( \frac{4x^2 - 1}{3} \right)^2 + 3x = 1$$

$$4 \frac{64x^4 - 32x^2 + 4}{9} + 3x = 1 \quad | \cdot 9$$

$$64x^4 - 32x^2 + 4 + 27x = 9$$

$$64x^4 - 32x^2 + 24x - 5 = 0$$

andere Lösung?

Andere Lösungen gehen nicht, wenn man die Gleichung dividiert.

$$(64x^4 - 32x^2 + 24x - 5) : (x+1) = 64x^3 - 64x^2 + 32x - 5$$

$$\begin{array}{r} 64x^4 - 64x^3 \\ \hline 64x^3 - 32x^2 + 24x - 5 \\ + 64x^3 + 64x^2 \\ \hline 0 + 32x^2 + 27x - 5 \\ - 32x^2 - 32x \\ \hline 0 + 5x - 5 \\ + 5x + 5 \\ \hline 0 \end{array}$$

Faktorisieren:  $(x+1)(64x^3 - 64x^2 + 32x - 5) = 0$

$$x^3 - x^2 + \frac{1}{2}x - \frac{5}{64} = 0 \quad x = \frac{1}{4}$$

$$\frac{1}{64} - \frac{1}{16} + \frac{1}{8} - \frac{5}{64} = 0 \quad \text{Division durch } (x - \frac{1}{4}) \text{ od. die obere durch } (4x - 1)$$

$$x = 204 \quad (64x^3 - 64x^2 + 32x - 5) : (4x - 1) = 16x^2 - 12x + 5$$

$$\begin{array}{r} 64x^3 - 64x^2 + 32x - 5 \\ - 64x^3 + 16x^2 \\ \hline -48x^2 + 32x - 5 \\ 48x^2 - 12x \\ \hline 20x - 5 \\ 0 \end{array}$$

Faktorisierung

$$(x+1)(4x-1)(16x^2 - 12x + 5) = 0$$

$$16x^2 - 12x + 5 = 0$$

$$x_{3/4} = \frac{12 \pm \sqrt{144 - 320}}{32} = \frac{12 \pm \sqrt{-176}}{32} = \frac{12 \pm 4i\sqrt{11}}{32} = \frac{3 \pm i\sqrt{11}}{8} = \frac{3}{8} \pm \frac{\sqrt{11}}{8}i$$

$$y = \frac{4x^2 - 1}{3}$$

$$y_1 = 1$$

$$y_2 = -\frac{1}{4}$$

$$y_3 = \frac{-3 + i\sqrt{11}}{8}$$

$$y_4 = \frac{-3 - i\sqrt{11}}{8}$$



$$\begin{cases} \sqrt[3]{x} - \sqrt[3]{y} = 1 \\ x - y = 217 \end{cases} \quad \begin{cases} \sqrt[3]{x} = u \\ \sqrt[3]{y} = v \end{cases} \quad \begin{cases} u - v = 1 \\ u^3 - v^3 = 217 \end{cases}$$

$$L = \{(-512/-729), (729/512)\}$$

$$\begin{cases} u - v = 1 \\ u^3 - v^3 = 217 \end{cases}$$

$$u = 1 + v$$

$$\begin{matrix} u_1 = -8 & x_1 = (-8^3) = -512 \\ u_2 = 9 & x_2 = 9^3 = 729 \end{matrix}$$

$$\begin{aligned} (u^3 - v^3) : (u - v) &= 217 \\ (u - v)(u^2 + uv + v^2) &= 217 \\ (u^2 + uv + v^2) &= 217 \end{aligned}$$

$$\begin{aligned} 3v^2 + 3v + 1 &= 217 \\ 3v^2 + 3v - 216 &= 0 \\ v^2 + v - 72 &= 0 \end{aligned}$$

$$\begin{matrix} y_1 = -729 \\ y_2 = 512 \end{matrix}$$

$$(1+v)^2 + uv + v^2 = 217$$

$$\begin{aligned} 1 + 2v + v^2 + v + v^2 &= 217 \\ 1 + 2v + v^2 + v + v^2 + v^2 &= 217 \end{aligned}$$

$$v_{1/2} = \frac{-1 \pm \sqrt{1 + 288}}{2} = \frac{-1 \pm 17}{2} = \begin{matrix} -9 \\ 8 \end{matrix}$$

$$\begin{cases} \lg \frac{x}{y} = 1 \\ \frac{1}{\lg x} - \frac{1}{\lg y} = 1 \end{cases} \quad \begin{cases} \lg x - \lg y = -1 \\ \frac{1}{\lg x} - \frac{1}{\lg y} = 1 \end{cases} \quad \begin{matrix} \lg x = u \\ \lg y = v \end{matrix}$$

$$\begin{cases} u - v = -1 \\ \frac{1}{u} - \frac{1}{v} = 1 \end{cases} \quad u = v - 1 \quad \frac{1}{v-1} - \frac{1}{v} = 1$$

$$\begin{aligned} v - v + 1 &= v^2 - v \\ -v^2 + v + 1 &= 0 \\ v^2 - v - 1 &= 0 \end{aligned}$$

$$v_{1/2} = \frac{1 \pm \sqrt{5}}{2} \quad \begin{matrix} v_1 = 1.5 \\ v_2 = -0.5 \end{matrix} = \frac{1 \pm \sqrt{5}}{2}$$

$$\begin{matrix} x_1 = 10^{\frac{\sqrt{5}-1}{2}} & y_1 = 10^{\frac{1+\sqrt{5}}{2}} \\ x_2 = 10^{\frac{-\sqrt{5}-1}{2}} & y_2 = 10^{\frac{1-\sqrt{5}}{2}} \end{matrix}$$

$$u = v - 1 = \frac{1 \pm \sqrt{5}}{2} - \frac{2}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\lg \frac{x}{y} = 1 \Rightarrow \frac{x}{y} = 0.1$$

$$\text{wenn } \lg \frac{x}{y} = 1 \Rightarrow \lg x \cdot \lg y = 1$$

$$\begin{aligned} \lg y - \lg x &= \lg x \cdot \lg y \\ \lg \frac{y}{x} &= \lg x \cdot \lg y = 1 \end{aligned}$$

$$\lg 0.1 y \cdot \lg y = 1$$

$$[\lg 0.1 + \lg y] \lg y = 1$$

$$[-1 + \lg y] \lg y = 1$$

$$\lg y = u$$

$$\begin{aligned} (-1 + u)u &= 1 \\ -u + u^2 &= 1 \\ u^2 - u - 1 &= 0 \end{aligned}$$



119/63)

$$\begin{cases} x+y=35 \\ x^2+y^2=625 \end{cases} \quad x=35-y$$

$$2y^2 - 70y + 600 = 0$$

$$y^2 - 35y + 300 = 0$$

$$(35-y)^2 + y^2 = 625$$

$$1225 - 70y + 2y^2 = 625$$

$$2y^2 - 70y + 1225 = 625$$

$$y_{1/2} = \frac{35 \pm 5}{2}$$

$$x = 35 \pm 5 \quad \begin{matrix} x_1 = 30 \\ x_2 = 40 \end{matrix} \quad \begin{matrix} y_1 = 20 \\ y_2 = 15 \end{matrix} \quad \begin{matrix} x_1 = 15 \\ x_2 = 20 \end{matrix}$$

Gesuchte Zahlen: 15 und 20

119/64)

$$\begin{cases} xy = 255 \\ x^2 + y^2 = 514 \end{cases} \quad x = \frac{255}{y}$$

$$y_2 = \frac{514 \pm 64}{2}$$

$$\begin{matrix} u_1 = 289 \\ u_2 = 225 \end{matrix}$$

$$\frac{255^2}{y^2} + y^2 = 514$$

$$y_{1/2} = \pm 17$$

$$x_{1/2} = \pm 15$$

$$y^4 - 514y^2 + 65025 = 0 \quad y^2 = u$$

$$y_{3/4} = \pm 15$$

$$x_{3/4} = \pm 17$$

$$u^2 - 514u + 65025 = 0$$

Die Zahlen heißen:  $\pm 15$  und  $\pm 17$

119/68)

Summanden  
1. 2. 3.  
x y  $3\frac{1}{3}x$

$$\begin{cases} x + y + 3\frac{1}{3}x = 60 \\ x \cdot 3\frac{1}{3}x = 60y \end{cases} \quad y = 60 - 4\frac{1}{3}x$$

$$3\frac{1}{2}x^2 = 60(60 - 4\frac{1}{3}x)$$

$$3\frac{1}{3}x^2 = 3600 - 260x$$

$$3\frac{1}{3}x^2 + 260x - 3600 = 0$$

$$x_1 = 12$$

$$y_1 = 8$$

1. Summand	: 12,	-90
2.	: 8,	450
3.	: 40,	-300

$$x_2 = -90$$

$$y_2 = 450$$

$$x_{1/2} = \frac{-260 \pm 340}{6\frac{2}{3}}$$

$$119/69) \quad \frac{x+y}{2} = 17$$

$$\sqrt{x^2 + y^2} = 8$$

119/70)

$$x+y=3.5$$

$$xy=3$$

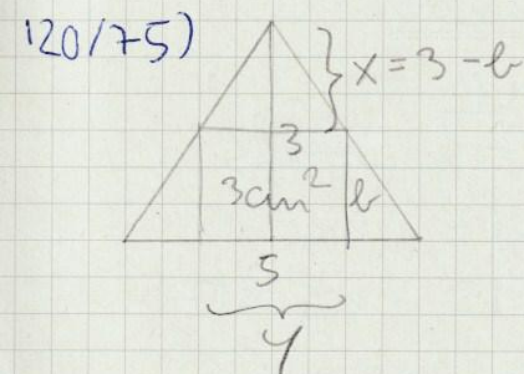
$$119/71) \quad x+y=80$$

$$xy=1500$$



120/72)  $x+y=23$   
 $x^2+y^2=289$

74 wenn a kleiner h = unmöglich



$$3-h : y = 3 : 5 \quad (3-x)y = 3$$

$$x : y = 3 : 5 \quad (3 - \frac{3y}{5})y = 3$$

$$5x = 3y \quad 3y - \frac{3y^2}{5} = 3$$

$$x = \frac{3y}{5} \quad 15y - 3y^2 = 15$$

$$+3y^2 - 15y + 15 = 0$$

$$y^2 - 5y + 5 = 0$$

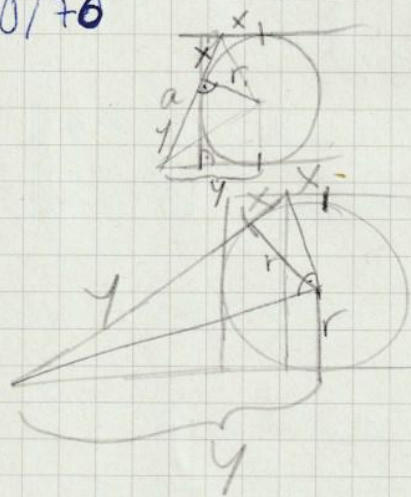
$$y_{1/2} = \frac{5 \pm \sqrt{5}}{2}$$

$$y_1 = \frac{5 + \sqrt{5}}{2} \quad x_1 =$$

$$y_2 = \frac{5 - \sqrt{5}}{2}$$

$$x_{1/2} = \frac{\frac{6}{2} \cdot \frac{5 \pm \sqrt{5}}{2}}{5} = \frac{30 \pm 6\sqrt{5}}{5} = \frac{30 \pm 6\sqrt{5}}{20} = \frac{15 \pm 3\sqrt{5}}{10}$$

120/76



$$x+y=a \quad x=a-y$$

$$(x+y)^2 = (y-x)^2 + 4r^2$$

$$a^2 = y^2 - 2yx + x^2 + 4r^2$$

$$a^2 = y^2 - 2y(a-y) + (a-y)^2 + 4r^2$$

$$a^2 = y^2 - 2ay - 2y^2 + a^2 - 2ay + y^2 + 4r^2$$

$$a^2 = 4r^2 - 2ay$$

$$4r^2 - 2ay = a^2$$

$$2ay - 4r^2 = -a^2$$

$$y = \frac{4r^2 - a^2}{2a}$$

$$x^2 + 2xy + y^2 = y^2 - 2xy + x^2 + 4r^2$$

$$xy = r^2$$

$$y(a-y) = r^2$$

$$y^2 - ay + r^2 = r^2$$

$$y^2 - ay = 0$$

$$y_{1/2} = \frac{ay \pm \sqrt{a^2y^2 - 4r^2}}{2}$$

$$y_{1/2} = \frac{a \pm \sqrt{a^2 - 4r^2}}{2}$$

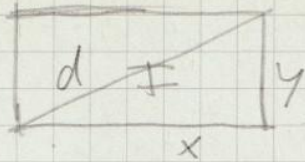
$$x = a - y$$

$$x = \frac{2a - a \pm \sqrt{a^2 - 4r^2}}{2} = \frac{a \pm \sqrt{a^2 - 4r^2}}{2}$$

$$x_{1/2} = \frac{a \pm \sqrt{a^2 - 4r^2}}{2}$$



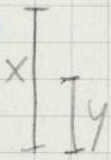
120/77)



$$xy = 660$$

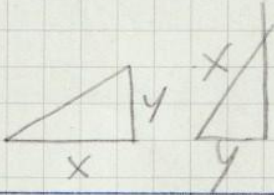
$$x^2 + y^2 = 3721$$

120/78)

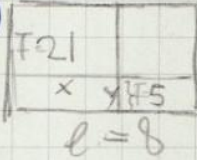


$$\begin{cases} x^2 + y^2 = 169 & + \\ x^2 - y^2 = 119 & + \end{cases}$$

$$\begin{aligned} 2x^2 &= 288 \\ x &= 12 \end{aligned}$$



120/79)



$$\begin{cases} x(6-y) = 21 & |6x - xy = 21| + \\ y(8-x) = 5 & |8y - xy = 5| - \end{cases}$$

$$6x - 8y = 16 \quad x = \frac{16 + 8y}{6} = \frac{8 + 4y}{3}$$

$$\frac{8 + 4y}{3}(6 - y) = 21$$

$$4y^2 - 16y + 15 = 0$$

$$(8 + 4y)(6 - y) = 63$$

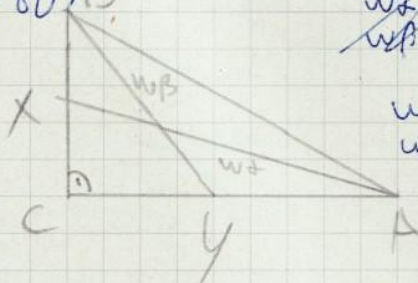
$$y_{1/2} = \frac{16 \pm 4}{8} = \frac{4 \pm 1}{2} \quad \begin{aligned} y_1 &= 2\frac{1}{2} & x_1 &= 6 \\ y_2 &= 1\frac{1}{2} & x_2 &= 4\frac{2}{3} \end{aligned}$$

$$48 - 8y + 24y - 4y^2 = 63$$

$$48 + 16y - 4y^2 = 63$$

$$-4y^2 + 16y - 15 = 0$$

120/80) 13



~~wa = 8m~~  
wb = 4m

wa = 11m  
wb = 8m

$$\begin{cases} x^2 + (\frac{y}{2})^2 = 64 \\ (\frac{x}{2})^2 + y^2 = 121 \end{cases}$$

$$\begin{cases} 4x^2 + y^2 = 256 \\ x^2 + 4y^2 = 484 \cdot 4 \end{cases}$$

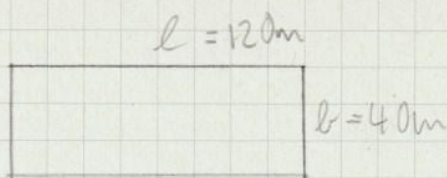
$$-15y^2 = -1690$$

$$\begin{aligned} y^2 &= 112 \\ y &= \pm 4\sqrt{7} \end{aligned}$$

81/82/83



121/81)



$$U = 320m \quad F = 4800m^2$$

$$l \cdot b = F$$

$$\begin{cases} (l-x)(l-y) = F \\ 2(x+y) = 40 \end{cases} \quad x = 20 - y$$

$$22800 = \underbrace{\sqrt{2} \cdot 2}_{2} \cdot \underbrace{\sqrt{2} \cdot 2}_{2} \cdot \underbrace{\sqrt{5} \cdot 5 \cdot 3 \cdot 19}_{5}$$

$$(l - 20 - y)(l - y) = F$$

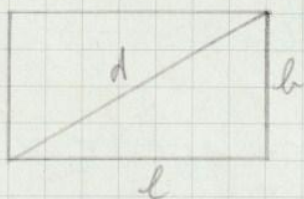
$$(100 - y)(40 - y) = F$$

$$4000 - 140y + y^2 = 4800$$

$$y^2 - 140y - 800 = 0$$

$$y_{1/2} = \frac{140 \pm 20\sqrt{57}}{2}$$

121/84)



$$l^2 + b^2 = 169$$

$$(l-4)^2 + (b+10)^2 = 289$$

$$l^2 - 8l + 16 + b^2 + 20b + 100 = 289$$

$$l^2 - 8l + b^2 + 20b = 173$$

$$\begin{array}{l} l^2 + b^2 = 169 \\ l^2 - 8l + b^2 + 20b = 173 \end{array} \quad \begin{array}{l} + \\ - \end{array}$$

$$8l - 20b = -4$$

$$5b = 1 + 2l$$

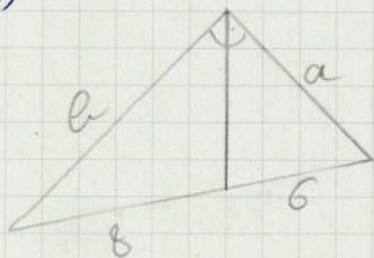
$$\underline{b = \frac{1+2l}{5}}$$

$$l^2 + \frac{(1+2l)^2}{25} = 169$$

$$25l^2 + 1 + 4l + 4l^2 = 4225$$

$$29l^2 + 4l$$

121/85)



$$\frac{a}{b} = \frac{6}{8} \quad \frac{a}{6} = \frac{b}{8} \quad \rightarrow a = \frac{3b}{4}$$

$$a^2 + b^2 = 196$$

$$9b^2 + 16b^2 = 3136$$

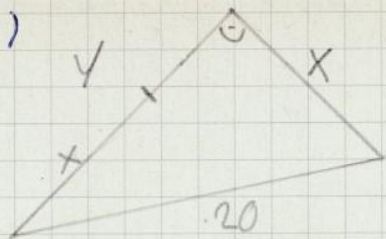
$$b = \frac{\sqrt{3136}}{5}$$

$$\underline{b = 11.2}$$

$$\underline{a = 8.4}$$



121/86)



$$y = x = x : (y - x)$$

$$x^2 + y^2 = 400$$

$$\rightarrow \frac{y}{x} = \frac{x}{y-x}$$

$$y(y-x) = x^2$$

$$y^2 - xy - x^2 = 0$$

$$x_{1/2} = \frac{-y \pm \sqrt{y^2 + 4x^2}}{2} = \frac{-y \pm y\sqrt{5}}{2} \quad x = y \frac{\sqrt{5}-1}{2}$$

$$\left(\frac{\sqrt{5}-1}{2}\right)^2 + y^2 = 400$$

$$y^2 \frac{3-\sqrt{5}}{2} + y^2 = 400$$

$$y^2 \left(\frac{3-\sqrt{5}}{2} + 1\right) = 400$$

$$\rightarrow y^2 \left(\frac{5-\sqrt{5}}{2}\right) = 400$$

$$y^2 = \frac{800}{5-\sqrt{5}}$$

$$y = \sqrt{\frac{800}{5-\sqrt{5}}} = \underline{\underline{17.013016}}$$

$$x = \underline{\underline{10.514622}}$$

121/87)



$$xy = 480 \quad x = \frac{480}{y}$$

$$x^2 + y^2 = (x+y-12)^2 \quad (x+y-12)(x+y-12)$$

$$\cancel{x^2} + \cancel{y^2} = \cancel{x^2} + \cancel{xy} - 12x + \cancel{xy} + \cancel{y^2} - 12y - 12x - 12y + 144$$

$$2xy - 24x - 24y + 144 = 0$$

$$xy - 12x - 12y + 72 = 0$$

$$480 - \frac{12 \cdot 480}{y} - 12y + 72 = 0$$

$$480y - 5760 - 12y^2 + 72y = 0$$

$$40y - y^2 - 480 + 6y = 0$$

$$-y^2 + 46y - 480 = 0$$

$$y^2 - 46y + 480 = 0$$

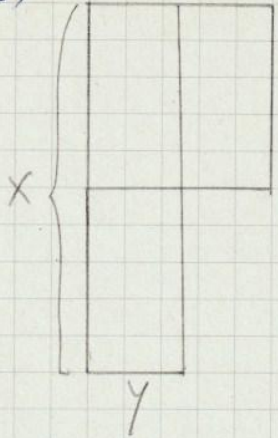
$$y_{1/2} = \frac{46 \pm 14}{2} = \begin{cases} 30 \\ 16 \end{cases}$$

$$x_1 = 16 \quad \parallel \quad x_2 = 30$$

$$y_1 = 30 \quad \parallel \quad y_2 = 16$$



121/88)



$$\begin{cases} 2(x+y) = 8 \\ xy = 1 \end{cases} \quad x = \frac{1}{y}$$

$$\begin{aligned} 2\frac{1}{y} + 2y &= 8 \\ \frac{2}{y} + 2y - 8 &= 0 \quad | \cdot y \end{aligned}$$

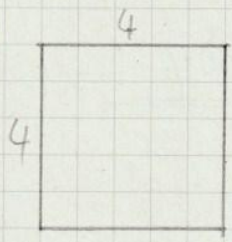
$$\begin{aligned} x_1 &= 0.26794919 & x_2 &= 3.7320508 \\ y_1 &= 3.7320508 & y_2 &= 0.26794919 \end{aligned}$$

$$\begin{aligned} 2 + 2y^2 - 8y &= 0 \\ 2y^2 - 8y + 2 &= 0 \end{aligned}$$

$$y_{1/2} = \frac{4 \pm 2\sqrt{3}}{2} = \underline{\underline{2 \pm \sqrt{3}}}$$

$$\begin{aligned} y^2 - 4y + 1 &= 0 \\ y_{1/2} &= \frac{4 \pm 2\sqrt{12}}{2} = \begin{cases} 3.7320508 \\ 0.26794919 \end{cases} \end{aligned}$$

121/89)



$$\begin{cases} xy = 8 \\ 2(x+y) = 16 \end{cases} \quad x = \frac{8}{y}$$

$$2\frac{8}{y} + 2y = 16$$

$$\begin{aligned} x_1 &= 1.1715729 & x_2 &= 6.8284271 \\ y_1 &= 6.8284271 & y_2 &= 1.1715729 \end{aligned}$$

$$\frac{16}{y} + 2y - 16 = 0$$

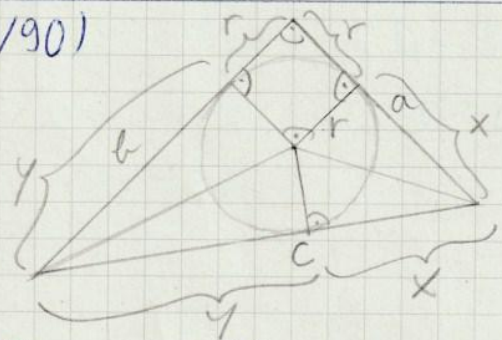
$$8 + y^2 - 8y = 0$$

$$y^2 - 8y + 8 = 0$$

$$y_{1/2} = \frac{8 \pm \sqrt{32}}{2} = \begin{cases} 6.8284271 \\ 1.1715729 \end{cases}$$

$$\frac{8 \pm 4\sqrt{2}}{2} = \underline{\underline{4 \pm 2\sqrt{2}}}$$

121/90)



$$a + b + c = 30 \text{ cm}$$

$$r = 2 \text{ cm}$$

$$\begin{cases} 2x + 2y = 26 \\ rx + ry + r^2 = \frac{(y+r)(r+x)}{2} \end{cases}$$

$$2(rx + ry + r^2) = (yr + yx + r^2 + rx)$$

$$2y = \frac{26}{2x}$$

$$\cancel{2rx} + \cancel{2ry} + r^2 = \cancel{yr} + yx + r^2 + \cancel{rx}$$

$$y = \frac{13}{2x}$$

$$r^2 - yx + rx + ry = 0$$

$$\frac{r^2 - \frac{13}{2} + \frac{13rx}{2x} + \dots}{2} = 0$$

$$r^2 - \frac{13}{2} + rx + \frac{13r}{2x} = 0$$

$$\cancel{r^2 - \frac{13}{2}}$$

$$4 - \frac{13}{2} + 2x + \frac{26}{2x} = 0$$

$$| \cdot x \quad -2.5x + 2x^2 + 13 = 0$$

$$4 - \frac{13}{2} + 2x + \frac{26}{2x} = 0$$

$$2x^2 - 2.5x + 13 = 0$$

$$| \cdot \frac{3}{x}$$



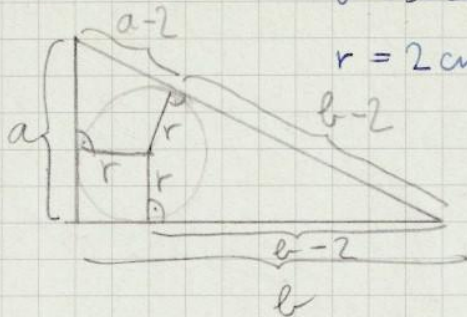
$$x_{1/2} = \dots$$

$$U = 30 \text{ cm}$$

$$r = 2 \text{ cm}$$

$$x^2 + y^2 = [30 - (x + y)]^2$$

5,12



$$\begin{array}{r} a + b + \sqrt{a^2 + b^2} = 30 \\ \sqrt{a^2 + b^2} = a + b - 4 \end{array} \quad \begin{array}{l} + \\ - \end{array}$$

$$\begin{array}{r} a + b = 34 - a - b \\ 2(a + b) = 34 \\ a + b = 17 \\ a = 17 - b \end{array}$$

$$\sqrt{(17 - b)^2 + b^2} = 17 - b + b - 4$$

$$\sqrt{289 - 34b + b^2 + b^2} = 14 - b$$

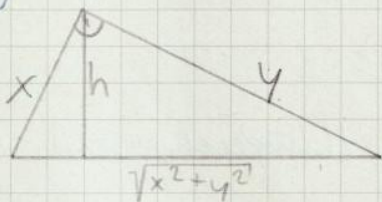
$$289 - 34b + 2b^2 = 196 - 28b + 16b^2$$

$$2b^2 - 34b + 93 = 0$$

$$\begin{array}{l} a_1 = 5 \\ b_1 = 12 \end{array} \quad \begin{array}{l} a_2 = 12 \\ b_2 = 5 \end{array}$$

$$b_{1/2} = \frac{34 \pm 14}{4} = \begin{array}{l} 12 \\ 5 \end{array}$$

122/91)



$$\begin{array}{l} h = 12 \text{ m} \\ U = 60 \text{ m} \end{array}$$

$$\begin{array}{r} x + y + \sqrt{x^2 + y^2} = 60 \\ \frac{xy}{2} = \frac{12 \cdot \sqrt{x^2 + y^2}}{2} \end{array} \quad \begin{array}{l} \cdot 12 \\ \cdot 2 \end{array}$$

$$xy = 12\sqrt{x^2 + y^2}$$

$$12x + 12y + 12\sqrt{x^2 + y^2} = 720$$

$$12x + 12y + xy = 720$$

$$12x + xy = 720 - 12y$$

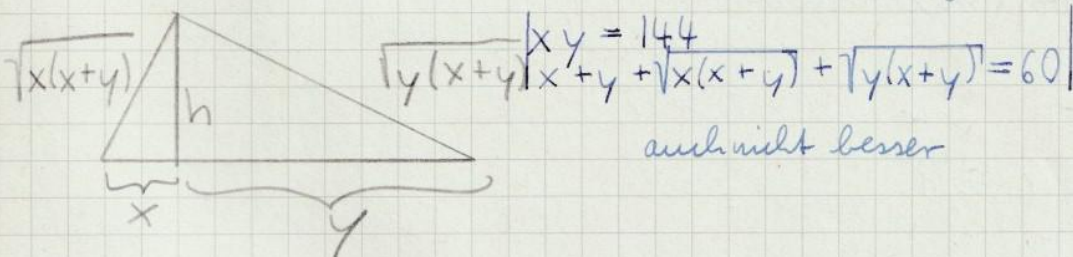
$$x(12 + y) = 720 - 12y$$

$$x = \frac{720 - 12y}{12 + y}$$

$$y \frac{720 - 12y}{12 + y} = 12\sqrt{\left(\frac{720 - 12y}{12 + y}\right)^2 + y^2}$$

$$y^2 \left(\frac{720 - 12y}{12 + y}\right)^2 = 144 \left[ \left(\frac{720 - 12y}{12 + y}\right)^2 + y^2 \right]$$

leichteren Weg suchen!



auch nicht besser

$$y^2 \left(\frac{720 - 12y}{12 + y}\right)^2 = 144 \left[ \left(\frac{720 - 12y}{12 + y}\right)^2 + y^2 \right]$$

$$\frac{518400y^2 - 17280y^3 + 144y^4}{(12 + y)^2} = \frac{74649600 - 2488320y + 20736y^2}{(12 + y)^2} + 144y^2$$

$$518400y^2 - 17280y^3 + 144y^4 = 74649600 - 2488320y + 20736y^2 + 20736y^2 + 3456y^3 + 144y^4$$

$$476928y^2 - 20736y^3 + 2488320y - 74649600 = 0 \quad | : 144$$

$$3312y^2 - 144y^3 + 17280y - 518400 = 0$$

$$23y^2 - y^3 + 120y - 3600 = 0$$

15,20



## Ungleichungen ersten Grades

$a < b$  ist kleiner als  $b$   $b - a > 0$

Def.  $a > b \Leftrightarrow a - b > 0$

Aussagen  
Ungleichheiten (numerische)  
 $\sqrt{37} < 5$  falsch  
 $3 \cdot 15 + 7 > 30$  richtig

Aussageformen (Ungleichungen)  
 $3x + 6 > 15$

### Eigenschaften

1) Eine Ungleichheit bleibt richtig (oder falsch), wenn man auf beiden Seiten die gleiche Zahl addiert (oder subtrahiert).

Beweis:

$a > b$ , d.h.  $a - b > 0$   
und  $c \in \mathbb{R}$

Behauptung:  $a + c > b + c$

Nach Def. gilt:  $a + c > b + c$  wenn  $a + c - (b + c) > 0$

Es ist aber  $a + c - (b + c) = a + c - b - c = a - b > 0$  nach Voraussetzung

2) Eine Ungleichheit bleibt richtig, wenn man beide Seiten mit der gleichen positiven Zahl multipliziert, (dividiert).



D.h. Voraussetzung  $a > b$  und  $c > 0$

Beh.  $ac > bc$

Beweis:  $ac - bc > 0 = c(a - b)$

$$\begin{matrix} c > 0 \\ a - b > 0 \end{matrix}$$

$$\Rightarrow c(a - b) > 0$$

3) Das Ungleichheitszeichen kehrt den Sinn, wenn man beide Seiten einer Ungleichung mit einer neg. Zahl multipliziert.

Voraussetzung:  $a > b$   $\wedge$   $c < 0$   
und

Behauptung:  $ac < bc$

Beweis: Die Beh. bedeutet, daß  $bc - ac > 0$

$$c(b - a) \quad \begin{matrix} c < 0 \\ b - a < 0 \end{matrix}$$

$$\Rightarrow c(b - a) = bc - ac > 0$$

125/4)  $x > 1$

5)  $u \neq$  zw.  $|u|$

6)  $\lg x^4 > \lg x$

$$4 \lg x > \lg x$$

$$\lg x > 0$$

125/32)  $2x - 3 > 3x - 1$

$$-2 > x$$

$$\underline{x < -2}$$

$$L = \{x \mid x \in \mathbb{R} \wedge x < -2\}$$

125/33)  $\frac{3}{2}x + 1 > x - 2$

$$\frac{3}{2}x + 3 > x$$

$$3x + 6 > 2x$$

$$x > -6$$

$$\underline{L = \{x \mid x \in \mathbb{R} \wedge x > -6\}}$$



$$126/40) ax - b > cx - d$$

$$ax - cx > b - d$$

$$x(a-c) > b-d \quad a \neq c$$

$$\underline{\underline{x > \frac{b-d}{a-c}}}$$

$$a > c$$

$$\underline{\underline{x < \frac{b-d}{a-c}}}$$

$$a < c$$

$$126/44) \left| \begin{array}{l} \frac{3}{4}x + 3 > 0 \\ x - 2 > 0 \end{array} \right| \left| \begin{array}{l} x + 4 > 0 \\ x > 2 \end{array} \right|$$

$$x > -4$$

$$x > 2$$

$$\underline{\underline{x > 2}}$$

$$126/45) \left| \begin{array}{l} -\frac{2}{3}x - 4 > 0 \\ -\frac{1}{2}x + 2 > 0 \end{array} \right| \left| \begin{array}{l} -2x - 12 > 0 \\ -x + 4 > 0 \end{array} \right| \rightarrow \left| \begin{array}{l} -x > 6 \\ -x > -4 \end{array} \right| \left. \begin{array}{l} x < -6 \\ x < 4 \end{array} \right\} \underline{\underline{x < -6}}$$

$$126/46) \left| \begin{array}{l} \frac{1}{2}x + 5 > -3x - 2 \\ \frac{1}{2}x - 1 < x - 2 \end{array} \right| \left| \begin{array}{l} x + 10 > -6x - 4 \\ x - 2 < x - 4 \end{array} \right| \rightarrow \left| \begin{array}{l} 7x > -14 \\ -x < -2 \end{array} \right| \rightarrow \left| \begin{array}{l} x > -2 \\ x > 2 \end{array} \right| \left. \right\} \underline{\underline{x > 2}}$$

$$126/47) \left| \begin{array}{l} x + 1 > \frac{1}{2}x - 2 \\ x + 1 < \frac{1}{3}x - 3 \end{array} \right| \left| \begin{array}{l} 2x + 2 > x - 4 \\ 3x + 3 < x - 9 \end{array} \right| \rightarrow \left| \begin{array}{l} x > -6 \\ 2x < -12 \end{array} \right| \rightarrow \left| \begin{array}{l} x > -6 \\ x < -6 \end{array} \right| \left. \right\} \underline{\underline{\text{keine Lösung}}}$$

$$126/48) (x-1)(x-6) > 0$$

$$1) \text{entweder: } x-1 > 0 \wedge x-6 > 0$$

$$2) \text{oder: } x-1 < 0 \wedge x-6 < 0$$

$$1) \left. \begin{array}{l} x > 1 \\ x > 6 \end{array} \right\} \underline{\underline{x > 6}} \quad 2) \left. \begin{array}{l} x < 1 \\ x < 6 \end{array} \right\} \underline{\underline{x < 1}}$$

$$\underline{\underline{L = \{x \mid x < 1 \vee x > 6\}}}$$

$$126/49) (2x-1)(3x+5) > 0$$

$$1) \text{entweder: } 2x-1 > 0 \wedge 3x+5 > 0$$

$$2) \text{oder: } 2x-1 < 0 \wedge 3x+5 < 0$$

$$1) \left. \begin{array}{l} 2x > 1 \\ x > \frac{1}{2} \end{array} \right\} \left. \begin{array}{l} 3x > -5 \\ x > -\frac{5}{3} \end{array} \right\} \Rightarrow \underline{\underline{x > \frac{1}{2}}} \quad 2) \left. \begin{array}{l} 2x < 1 \\ x < \frac{1}{2} \end{array} \right\} \left. \begin{array}{l} 3x < -5 \\ x < -\frac{5}{3} \end{array} \right\}$$

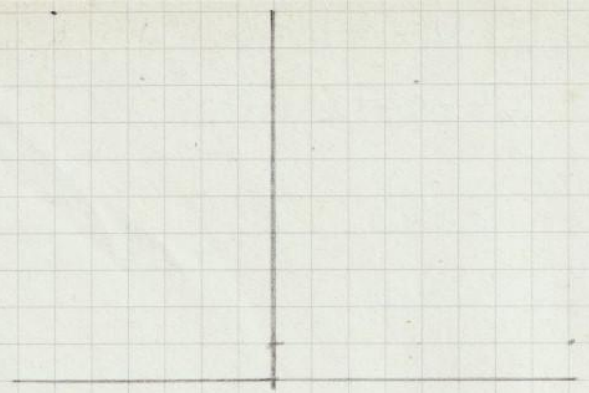
$$\underline{\underline{L = \{x \mid x < -\frac{5}{3} \vee x > \frac{1}{2}\}}}$$

$$\underline{\underline{x < -\frac{5}{3}}}$$



49)  $y = ax^2 + b \cdot x + c$   
 $y = 6x^2 + 7x - 5$

$y = 12x - 7$   
 $x = \frac{1}{2}$



126/51)  $(2-3x)(2x+5) > 0$

1)  $2-3x > 0 \wedge 2x+5 > 0$

$2 > 3x \wedge 2x > -5$   
 $x < \frac{2}{3} \wedge x > -\frac{5}{2}$

2)  $2-3x < 0 \wedge 2x+5 < 0$

$2 < 3x \wedge 2x < -5$   
 $x > \frac{2}{3} \wedge x < -\frac{5}{2}$

$\frac{2}{3} < x < -\frac{5}{2}$   
 $x < -\frac{5}{2}$

$x > -\frac{5}{2}$   $L = \{x | x > -\frac{5}{2} \wedge x < \frac{2}{3}\}$

126/52)  $(x+3)(x-4) < 0$

1)  $(x+3) < 0 \wedge x-4 < 0$

$x < -3 \wedge x < 4$   
 $x < -3$

2)  $x+3 > 0 \wedge x-4 > 0$

$x > -3 \wedge x > 4$   
 $x > 4$

$L = \{x | x < -3 \vee x > 4\}$

126/54)  $(2x-1)(3x+5) < 0$

1)  $2x-1 > 0 \wedge 3x+5 > 0$

$2x > 1 \wedge 3x > -\frac{5}{3}$   
 $x > \frac{1}{2} \wedge x > -\frac{5}{3}$   
 $x > \frac{1}{2}$

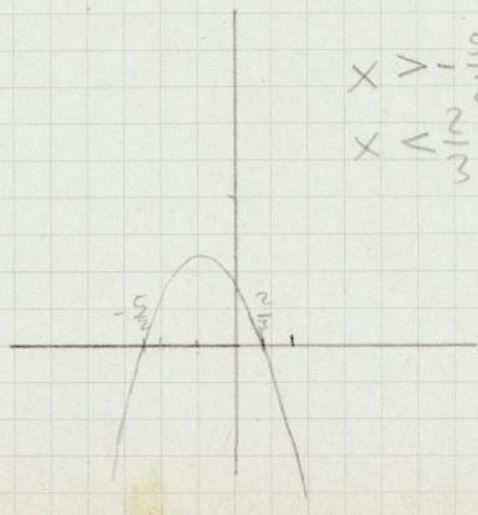
2)  $2x-1 < 0 \wedge 3x+5 < 0$

$2x < 1 \wedge 3x < -\frac{5}{3}$   
 $x < \frac{1}{2} \wedge x < -\frac{5}{3}$   
 $x < -\frac{5}{3}$

$L = \{x | x > \frac{1}{2} \vee x < -\frac{5}{3}\}$

51)  $(2-3x)(2x+5) > 0$   
 $4x - 6x^2 + 10 - 15x > 0$   
 $-6x^2 - 11x + 10 > 0$

$x > -\frac{5}{2}$   
 $x < \frac{2}{3}$



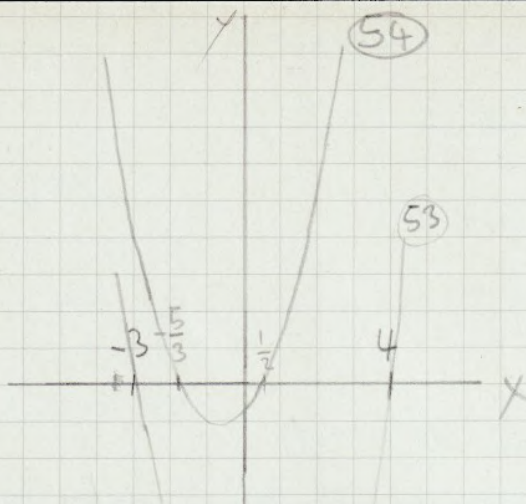


$$53) (x+3)(x-4) < 0$$

$$x^2 - x - 12 < 0$$

$$x > -3$$

$$x < 4$$



$$54) (2x-1)(3x+5) < 0$$

$$6x^2 + 7x - 5 < 0 \quad x > -\frac{5}{3}$$

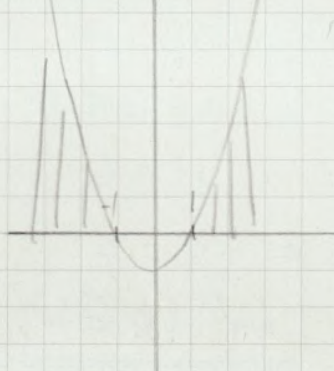
$$x < \frac{1}{2}$$



$$126/58) \frac{x-1}{x+1} > 0 \quad | (x+1)^2$$

$$\frac{(x-1)(x+1)}{x^2-1} > 0$$

$$\underline{x < -1 \vee x > 1}$$



$$126/59) \frac{x+1}{x-2} < 0 \quad | (x-2)^2$$

$$(x+1)(x-2) < 0$$

$$\underline{x^2 - x - 2 < 0}$$



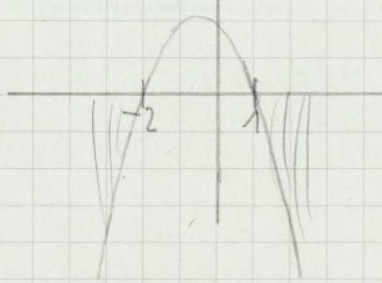
$$60) \frac{1-x}{x+2} < 0 \quad | (x+2)^2$$

$$(1-x)(x+2) < 0$$

$$x+2 - x^2 - 2x$$

$$-x^2 - x + 2 < 0$$

$$x < -2 \vee x > 1$$





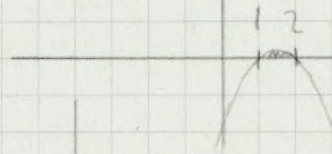
$$127/6(1) \quad \frac{2-x}{x-1} > 0 \quad | \cdot (x-1)^2$$

$$(2-x)(x-1) > 0$$

$$2x - 2 - x^2 + x > 0$$

$$-x^2 + 3x - 2 > 0$$

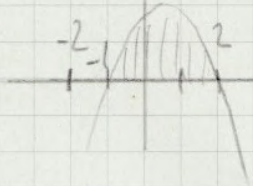
$$\underline{x > 1 \wedge x < 2}$$



$$127/1) \quad -x^2 + x + 2 > 0$$

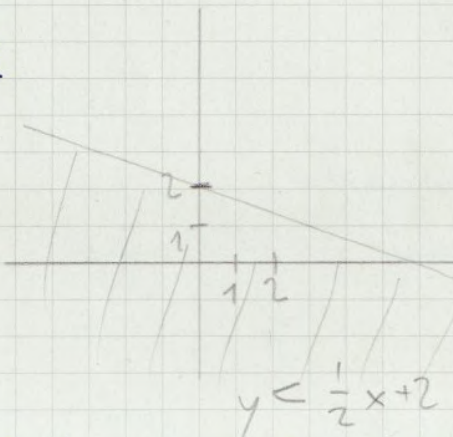
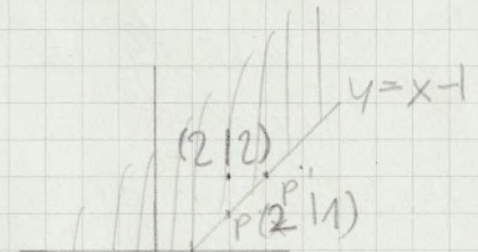
$$(2-x)(x+1) > 0$$

$$\underline{2 > x > -1}$$



$$127/6(8) \quad a) \quad y > x - 1$$

$$b) \quad y < -\frac{1}{2}x + 2$$



72, 2, 3, 4

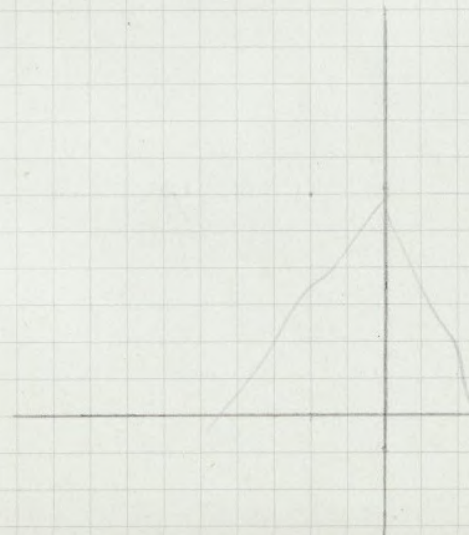
$$127/72) \quad \begin{cases} 2x + y - 6 < 0 \\ 6x - 5y + 30 > 0 \\ y + 1 > 0 \end{cases}$$

$$y < -2x + 6$$

$$-y > -\frac{6x + 30}{5}$$

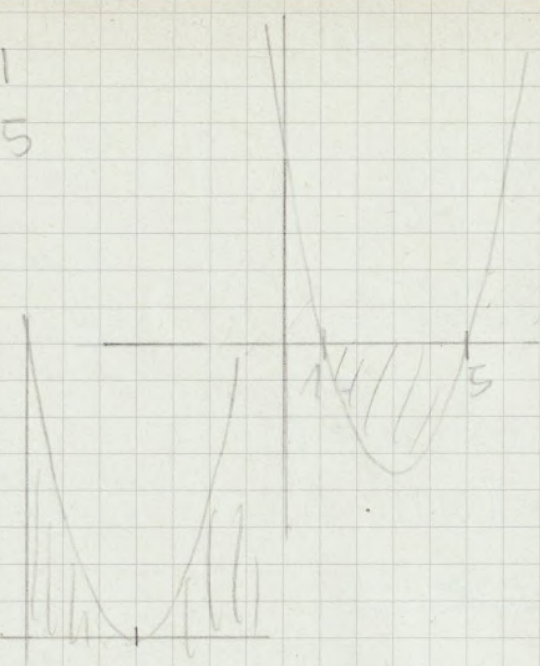
$$y > -1$$

$$y < \frac{6x + 30}{5}$$

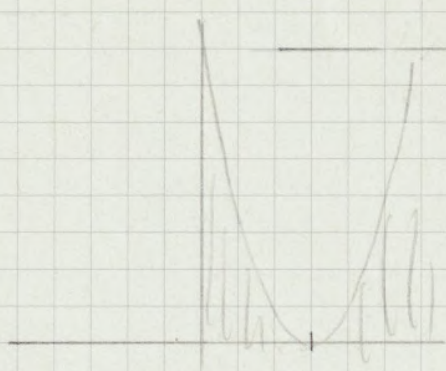




127/2)  $x^2 - 6x + 5 < 0$   
 $(x-1)(x-5) < 0$   $x < 1$   
 $x < 5$   
 $L = \{x | x > 1 \wedge x < 5\}$



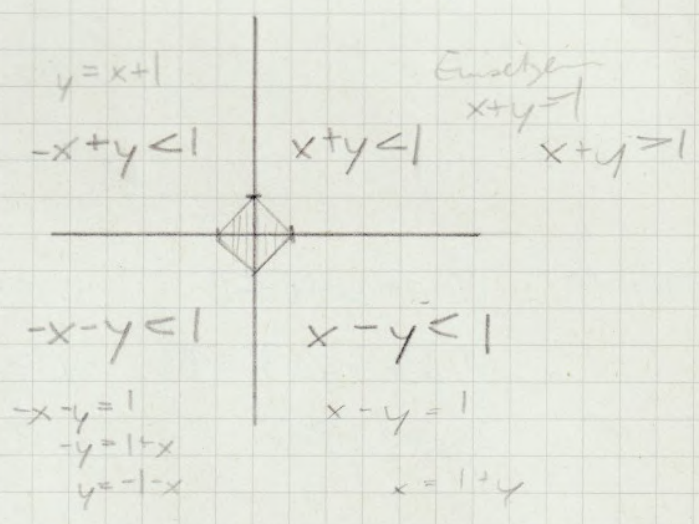
127/3)  $x^2 - 6x + 9 > 0$   
 $(x-3)(x-3) > 0$   
 $x > 3$   $x < 3$



127/4)  $x^2 - 2x + 2 < 0$   
 $(x-1)(x-2)$   
 $(x^2 - 2x + 1) + 1$   
 $(x-1)^2 + 1$  ist immer positiv!  
 → keine Lösung

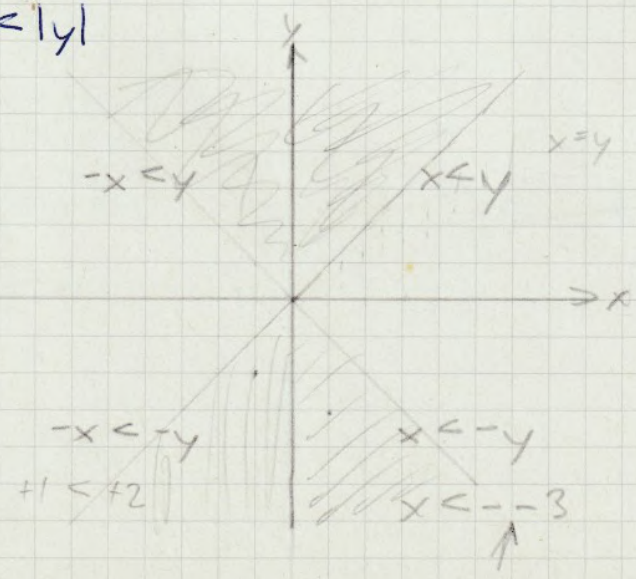
127/74)  $|x| + |y| < 1$

$|x| \begin{cases} x, \text{ wenn } x \geq 0 \\ -x, \text{ wenn } x < 0 \end{cases}$



74: Fallunterscheidung

127/75)  $|x| < |y|$



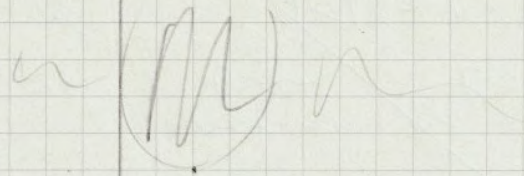


$$127/5) x^2 - 4x + 5 > 0$$

$$(x-2)^2 + 1 > 0$$

$$(x-2)(x-2) + 1$$

2



$$6) -x^2 - 2x - 1 < 0$$

$$-(x^2 + 2x + 1) < 0$$

$$-(x+1)^2 < 0 \quad x \neq -1$$

$$\underline{L = \{x \mid x \in \mathbb{R}, x \neq -1\}}$$

$$7) x^2 - 5x + 4 > 0$$

$$(x^2 - 5x + \frac{25}{4}) + 4 - \frac{25}{4} > 0$$

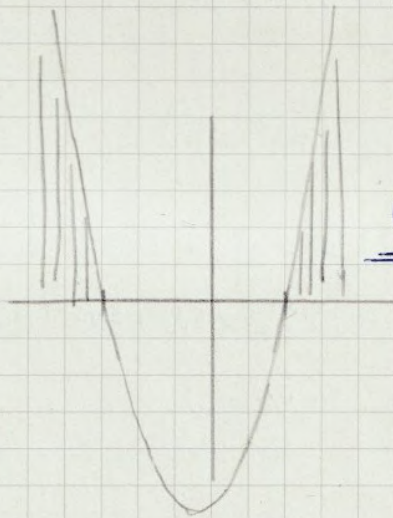
$$(x - \frac{5}{2})^2 - \frac{9}{4} > 0$$



$$8) x^2 + x - 6 > 0$$

$$(x+3)(x-2) = 0$$

$$\text{wenn } x = 2 \\ x = -3$$



$$\underline{L = \{x \mid x > 2 \vee x < -3\}}$$

9, 10, 11, 12

$$-x^2 + x - 12 > 0$$

$$-x^2 + x + \frac{1}{4} - \frac{1}{4}$$

$$x^2 - x + 12 < 0$$

$$x(x - \frac{1}{2})$$

$$(x^2 - x + 1) + 11 < 0$$



127/10)  $-x^2 + 7x > 0$

$-x(x - 7) > 0$

1)  $-x > 0$        $-x < 0$   
 $x - 7 > 0$        $x - 7 < 0$

$x < 0$  }  $x > 7$        $x > 0$  }  $x < 7$   
 $x > 7$  }  $x < 7$

$-x(x - 7) > 0$

$-[x(x - 7)] > 0$

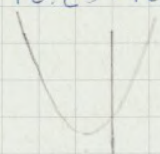
$x < 0 \vee x > 7$        $x \text{ zw. } 0 \text{ + } 7$

$-(x + 3.5)^2 - 12.25 > 0$

$-(x^2 + 7.5x + 12.25) - 12.25 > 0$

$-x^2 - 7.5x - 12.25 + 12.25 > 0$

keine Lösung



128/11)  $x^2 + 9x < 0$        $x(x + 9) < 0$

$x^2 + 9x + 20.25 < 0$        $x < 0 \wedge x > 9$

$(x + 4.5)^2 - 20.25 < 0$        $x \text{ zw. } -9 \text{ + } 0$

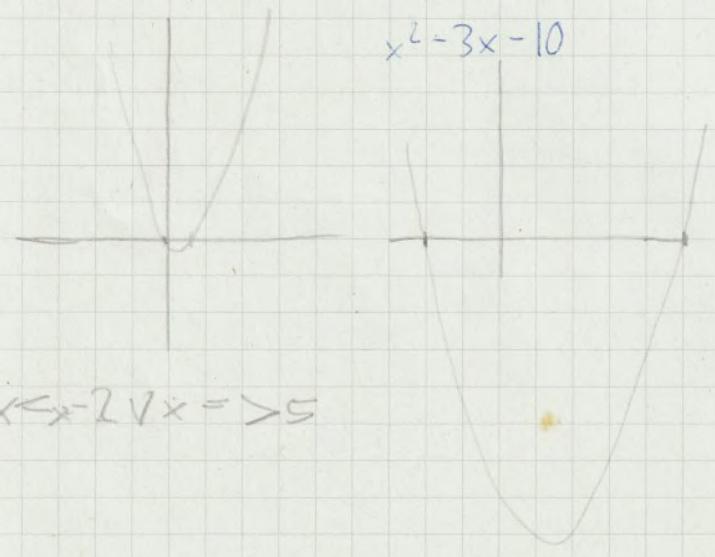
12)  $3x^2 - 5x + 7 > 0$        $x > 2 - \frac{6.25}{9}$

$x^2 - \frac{5}{3}x + \frac{6.25}{9} + 7 - \frac{6.25}{9} > 0$

$(x - \frac{2.5}{3})^2 + 7 - \frac{6.25}{9} > 0$        $x > 0$

128/19)  $x(2x - 1)(x + 2)(x - 5) > 0$

entweder  $x(2x - 1) \wedge (x + 2)(x - 5) > 0$  oder  $x(2x - 1) \wedge (x - 5) < 0$



$x < -2 \vee x > 5$

$x > 0$   
 $x < \frac{1}{2}$

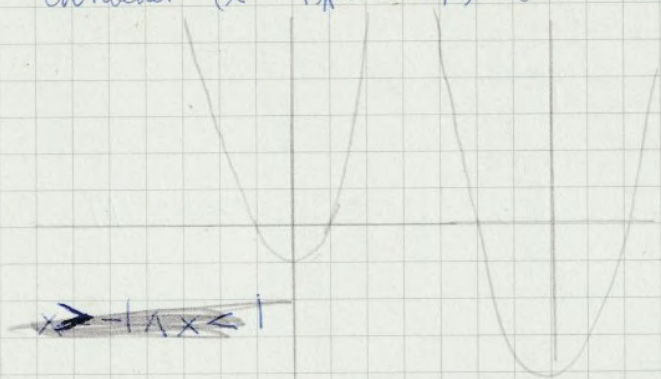
$L = \{x | x < -2 \vee x > 5 \vee 0 < x < \frac{1}{2}\}$



$$128/20) (x^2 - 1)(x^2 - 4) > 0$$

$$\text{entweder } (x^2 - 1) \wedge (x^2 - 4) > 0$$

$$\text{oder } (x^2 - 1) \wedge (x^2 - 4) < 0$$



$$-2 < x < 2$$

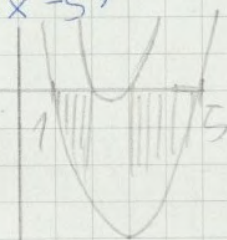
$$L = \{x \mid x < -2 \vee 2 < x \vee -1 < x < 1\}$$

$$\text{oder } (x^2 - 1) < 0 \wedge (x^2 - 4) < 0$$

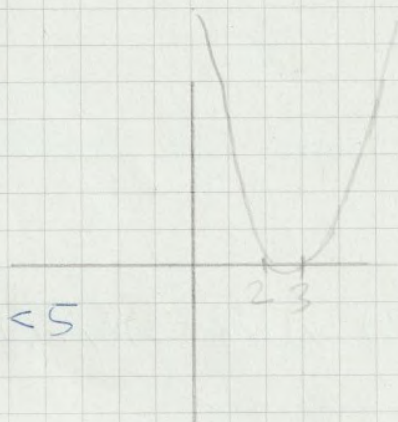
$$x > -1 \wedge x < 1$$

$$128/22) (x^2 - 6x + 5)(x^2 - 5x + 6) < 0$$

$$\text{entweder: } \begin{matrix} x^2 - 6x + 5 < 0 & \wedge & x^2 - 5x + 6 > 0 \\ (x-1)(x-5) & & (x-3)(x-2) \end{matrix}$$



$$1 < x < 2 \vee 3 < x < 5$$



$$\text{oder: } x^2 - 6x + 5 > 0 \wedge x^2 - 5x + 6 < 0$$

Wann nie erfüllt werden

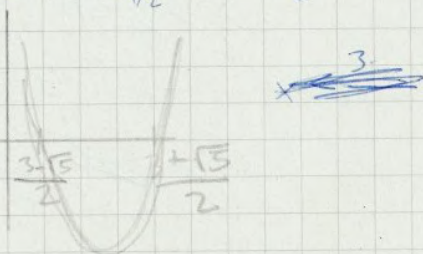
$$L = \{x \mid 1 < x < 2 \vee 3 < x < 5\}$$

$$128/24) x + \frac{1}{x} > 3$$

$$x^2 + 1 > 3x \text{ für } x > 0 \Rightarrow x^2 - 3x + 1 > 0$$

$$x^2 + 1 < 3x \text{ für } x < 0$$

$$x_{1/2} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$



$$0 < x < \frac{3 - \sqrt{5}}{2} \vee x > \frac{3 + \sqrt{5}}{2}$$

$x^2 - 3x + 1 < 0$  für  $x < 0$  keine Lösung

$$L = \left\{ x \mid 0 < x < \frac{3 - \sqrt{5}}{2} \vee x > \frac{3 + \sqrt{5}}{2} \right\}$$



$$128/25) \frac{2x+1}{1-x} > 2$$

$$1) 2x+1 > 2-2x \text{ wenn } 1-x > 0, x < 1$$

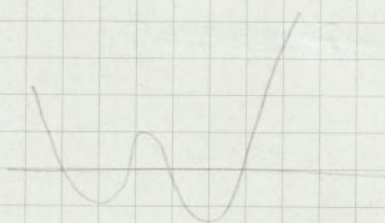
$$2) 2x+1 < 2-2x \text{ wenn } 1-x < 0, x > 1$$

$$1) 4x-1 > 0 \quad x > \frac{1}{4} \quad 2) 4x-1 < 0 \quad x < \frac{1}{4} \quad x > 1! \quad W$$

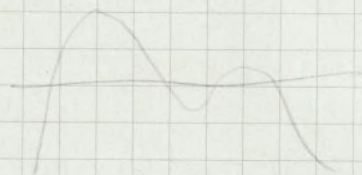
21, 23

$$L = \left\{ x \mid x > \frac{1}{4} \wedge x < 1 \right\}$$

$$\text{zu } \frac{1}{5} \quad 0 \quad x < -\frac{1}{4} \quad \text{zu } 0 \text{ u. } \frac{1}{5} \quad x > 2$$



23



siehe M. Ergänzungen

$$128/23) x(2x+3)(x-2)(1-x) > 0$$

entweder

$$x(2x+3) > 0 \quad \wedge \quad (x-2)(1-x) > 0$$

$$x > 0 \quad \wedge \quad 1 < x < 2$$

$$\vee \quad x < -\frac{3}{2}$$

$$\rightarrow 1 < x < 2$$

$$\text{oder } x(2x+3) < 0 \quad \wedge \quad (x-2)(1-x) < 0$$

$$-\frac{3}{2} < x < 0 \quad \wedge \quad 1 > x \vee x > 2$$

$$\rightarrow 0 > x > \frac{3}{2}$$

$$L = \left\{ x \mid 1 < x < 2 \vee 0 > x > \frac{3}{2} \right\}$$



128/31 a)  ~~$8 < x^2 - 14x + 50 < 26$~~

a)  $5 < x^2 - 14x + 50 < 26$

$4 < (x-7)^2 < 25$

entw.  $2 < x-7 < 5$

oder  $-2 < x-7 < -5$

$9 < x < 12$

$5 > x > 2$

$L = \{x \mid 9 < x < 12 \vee 5 > x > 2\}$

$x^2 - 14x + 50 = 26$

$x^2 - 14x + 50 = 5$

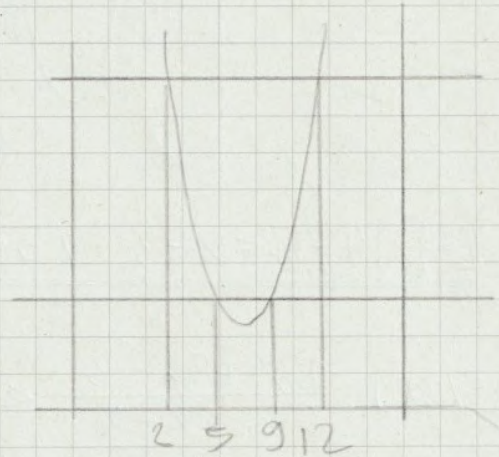
$x^2 - 14x + 24 = 0$

$x^2 - 14x + 45 = 0$

$(x-2)(x-12) = 0$

$(x-5)(x-9) = 0$

$x_1 = 5 \quad x_2 = 9$



128/31 b)  $0 < x^2 - 4x + 3 < 5$

$x^2 - 4x + 3 < 5$

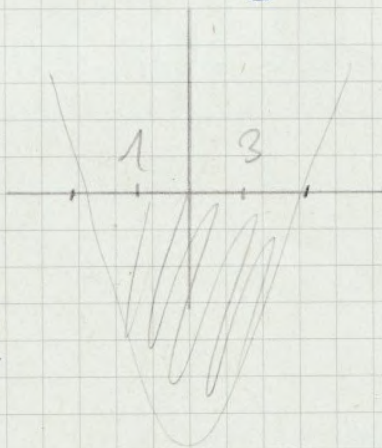
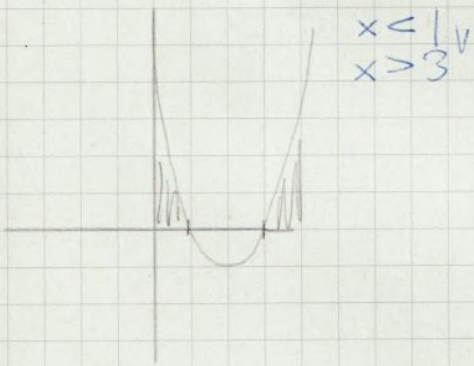
$x^2 - 4x + 3 > 0$   
 $(x-1)(x-3) > 0$

$x^2 - 4x - 2 < 0$

$x^2 - 4x - 2 = 0$

$x_{1/2} = \frac{4 \pm 2\sqrt{6}}{2}$

$x_{1/2} = 2 \pm \sqrt{6}$



$L = \{x \mid 2 - \sqrt{6} < x < 1 \vee 3 < x < 2 + \sqrt{6}\}$

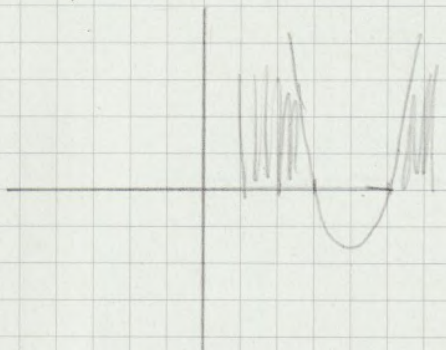


$$28/31c) 5 < x^2 - 48x + 20 < 19$$

$$x^2 - 48x + 20 > 5$$

$$x^2 - 48x + 15 > 5$$

$$(x-3)(x-5) > 0$$

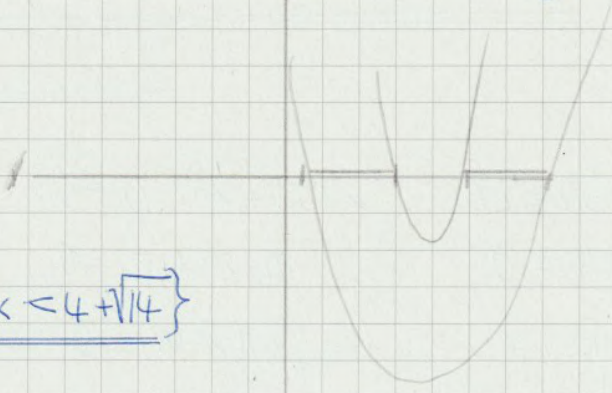


$$x^2 - 8x + 20 < 19$$

$$x^2 - 8x + 2 < 0$$

$$x^2 - 8x + 2 = 0$$

$$x_{1/2} = \frac{8 \pm \sqrt{64-8}}{2} = \frac{8 \pm 2\sqrt{14}}{2} = 4 \pm \sqrt{14}$$



$$\mathbb{L} = \{x \mid 4 - \sqrt{14} < x < 3 \vee 5 < x < 4 + \sqrt{14}\}$$

$$128/32) \frac{2x^2 + 5x - 3}{6x^2 - x - 2} < 0$$

$$1) 2x^2 + 5x - 3 < 0 \wedge 6x^2 - x - 2 > 0$$

$$x_{1/2} = \frac{-5 \pm 7}{4}$$

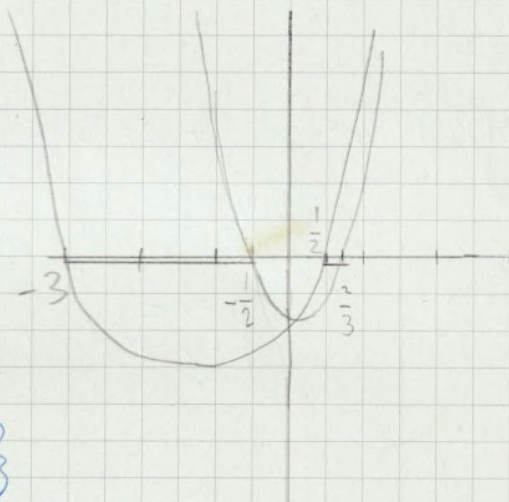
$$x_1 = 3 \frac{1}{2}$$

$$x_2 = -3$$

$$x_{3/4} = \frac{1 \pm 7}{12}$$

$$x_3 = \frac{2}{3}$$

$$x_4 = -\frac{1}{2}$$



$$\mathbb{L} = \left\{x \mid -3 < x < -\frac{1}{2} \vee \frac{1}{2} < x < \frac{2}{3}\right\}$$

$$128/30e) -\frac{7}{2x-4} > 1 + \frac{11}{2x-12}$$

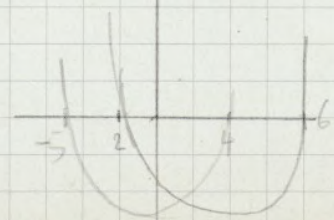
$$0 > 1 + \frac{11}{2x-12} + \frac{7}{2x-4}$$

$\hookrightarrow 2(x-6) \quad \hookrightarrow 2(x-2)$

$$0 > \frac{7(x-6) + 2(x-2)(x-6) + 11(x-2)}{2(x-2)(x-6)}$$

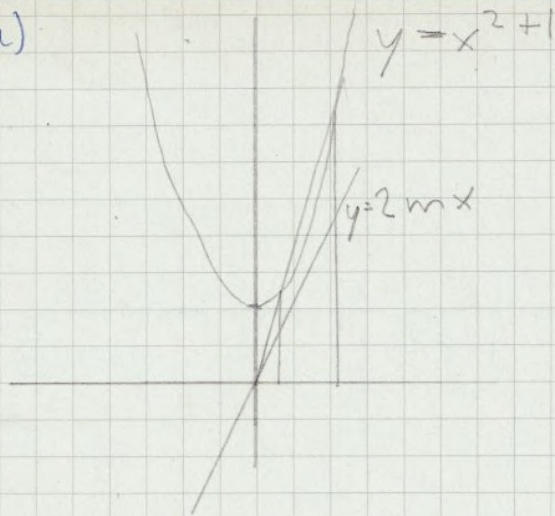
$$0 > \frac{2x^2 + 2x - 40}{2(x-2)(x-6)} = \frac{(x^2 + x - 20)}{x^2 - 8x + 12}$$

$$\mathbb{L} = \{x \mid -5 < x < 2 \vee 4 < x < 6\}$$





128/33) a)



$$x^2 + 1 = 2mx$$

$$x^2 + 1 > 2mx$$

$x^2 - 2mx + 1 = 0$  darf keine reelle Lösung haben

Die Diskriminante muß also negativ sein:

$$4m^2 - 4 < 0$$

$$m^2 < 1 \quad \underline{\underline{L = \{m \mid -1 < m < 1\}}}$$

b)  $x^2 + mx + 1$

$$m^2 - 4 < 0 \quad \underline{\underline{-2m < 2}}$$

c)  $m > 0$

$$mx^2 + 2(2m+1)x + 5m+4 > 0$$

$$4(2m+1)^2 - 4m(5m+4) < 0$$

$$16m^2 + 16m + 4 - 20m^2 - 16m < 0$$

$$\lim_{m > 0} | \dots | > 1 \rightarrow \underline{\underline{m > 1 \vee m = 0}}$$

128/34) Wenn Diskriminante = 0,  $\rightarrow$  zwei gleiche Lösungen

b) Diskriminante  $> 0 \in \mathbb{R}$

c) Diskriminante  $< 0 \in$  der komplexen Zahlen

a)  $(a+3)x^2 + 2ax + 4 = 0$

$$4a^2 - 16(a+3) = 0$$

$$4a^2 - 16a - 48$$

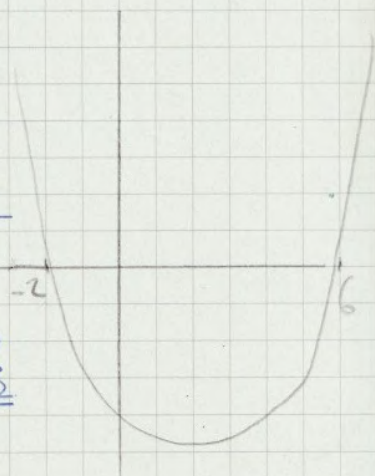
$$a^2 - 4a - 12 = 0$$

$$a_{1/2} = \frac{4 \pm 8}{2} \quad \left\langle \begin{array}{l} 6 \\ -2 \end{array} \right\rangle$$

b)  $a^2 - 4a - 12 > 0$

$$\underline{\underline{L = \{a \mid a > 6 \vee a < -2\}}}$$

c)  $\underline{\underline{L = \{a \mid -2 < a < 6\}}}$





128/35)

$$x^2 - 8x + a > 10 \quad x^2 - 8x - 10 > -a$$

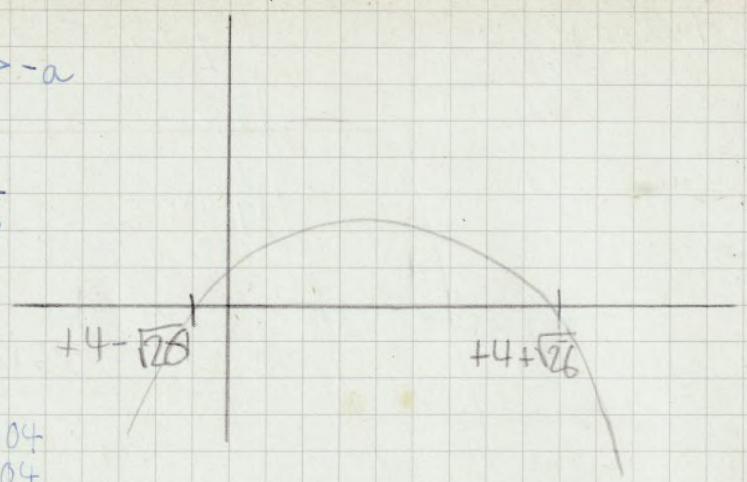
$$-x^2 + 8x + 10 \leq a$$

$$x_{1/2} = \frac{-8 \pm \sqrt{104}}{-2} = \frac{-8 \pm 2\sqrt{26}}{-2} = +4 \pm \sqrt{26}$$

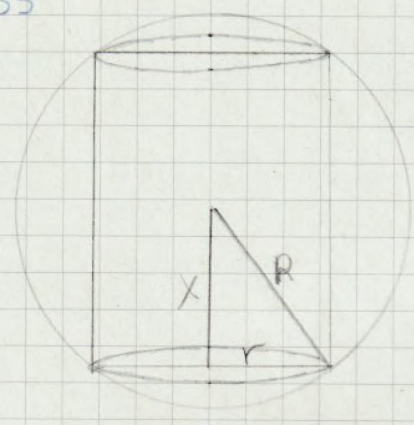
$$x^2 - 8x + a - 10 = 0$$

$$x_{1/2} = \frac{8 \pm \sqrt{64 - 4a + 40}}{2}$$

$$\begin{aligned} 104 - 4a &< 0 \\ -4a &< -104 \\ 4a &> 104 \\ \underline{a > 26} \end{aligned}$$



128/35\*



$$r = \sqrt{R^2 - x^2}$$

$$M = 2 \cdot \pi \cdot \sqrt{R^2 - x^2} \cdot 2x > 0$$

$$4\pi x \sqrt{R^2 - x^2} = k \cdot 4\pi R^2$$

$$x^2(R^2 - x^2) = k^2 R^4$$

$$-x^4 + R^2 x^2 + k^2 R^4 = 0$$

Lösungen, wenn Diskriminante (D)

$$D = R^4 - 4R^2 R^4 > 0$$

$$1 - 4k^2 \geq 0$$

$$\geq 4k^2$$

$$k^2 \leq \frac{1}{4} \quad \underline{k \geq 0 \wedge k \leq \frac{1}{2}}$$



m	D	$S = x_1 + x_2$	$P = x_1 x_2$	Art der Lösungen
$-\infty$ ↓	$D < 0$	1	negativ, wenn $m < 1$ 0, wenn $m = 1$ positiv, wenn $m > 1$	Die Gleichung hat für jeden Wert von m reelle Lösungen.
2	0	1	$\frac{1}{4}$	
	$D > 0$	1	positiv	
$+\infty$				

$-\infty < m < 1$ : Zwei verschiedene Lösungen ( $\pm$ )

$m = 1$ : eine der Lösungen ist 0

$m > 2$ : beide Lösungen sind positiv

$$m^2 x^2 - m^2 x + m - 1 = 0$$

a) immer reelle Wurzeln

$$D = m^4 - 4m^2 \cdot (m - 1) > 0$$

$$m^4 - 4m^3 + 4m^2 > 0$$

$$m^2(m^2 - 4m + 4) > 0$$

$$m^2(m - 2)^2 > 0$$

$$m \neq 2$$

Satz v. Vieta

$$S = -\frac{b}{a} = \frac{m^2}{m^2} = 1 \quad (\text{Summe der Lösungen})$$

$$P = \frac{c}{a} = \frac{m-1}{m^2} \quad (\text{Produkt der Lösungen})$$

38 wenn  $m=2 \rightarrow$  nur eine Lösung

$$2x^2 - 10mx + 7m - 1 = 0$$

$$D = 100m^2 - 8(7m - 1) \geq 0$$

$$100m^2 - 56m + 8 > 0$$

$$25m^2 - 14m + 2 > 0$$

$$m_{1,2} = \frac{14 \pm \sqrt{196 - 200}}{50} = \frac{14 \pm \sqrt{-4}}{50}$$

$$= \frac{7}{25} \pm \frac{2i}{50} = \frac{7}{25} \pm \frac{i}{25}$$



m	D	$S = -\frac{b}{a}$	$P = \frac{c}{a}$
$-\infty$	+	$5m < 0$	$\frac{7m-1}{2}$
$\frac{1}{7}$	+	$5m > 0$	0
0	+	0	$-\frac{1}{2}$
$+\infty$		$5m > 0$	$3\frac{1}{2}m - \frac{1}{2}$

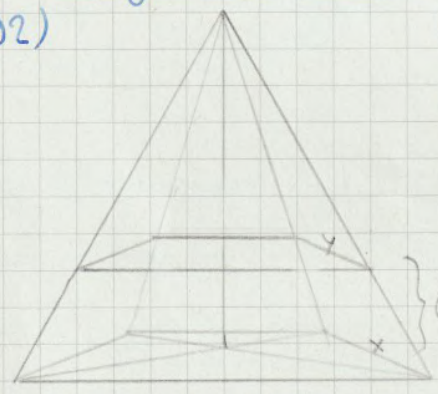
129/38)  $2x^2 - 10mx + 7m - 1 = 0$

$S = \frac{10m}{2} = 5m$

$P = \frac{c}{a} = \frac{7m-1}{2}$

Aufgaben aus der Stereometrie

122/92)



$h = 6 \text{ cm}$   
 $V = 56 \text{ cm}^3$   
 $x + y = 6 \text{ cm}$

$V = \frac{1}{3}(G + D + \sqrt{GD})$

$2(x^2 + xy + y^2) = 56$      $x = 6 - y$

$y_1 = 2$      $x_1 = 4$

$y_2 = 4$      $x_2 = 2$

$(6-x)^2 + x(6-y) + y^2 = 28$

$36 - 12y + y^2 + 6y - x^2 + y^2 = 28$

$y^2 - 6y + 8 = 0$

$(y-2)(y-4) = 0$



- 9) Berechne die Summe  $s_n$  der geometrischen Folge aus  $a_1$ ,  $q$  und  $a_n$ . [Setze in der Summenformel für  $q^{n-1} = \frac{a_n}{a_1}$ ]
- 10) Sind von den 5 Größen  $a_1$ ,  $q$ ,  $n$ ,  $a_n$ ,  $s_n$  einer geometrischen Folge drei gegeben, so lassen sich im allgemeinen die beiden andern hieraus berechnen. Wie viele und welche Aufgabenarten lassen sich so bilden? Welche zwei Aufgabenarten sind mit einfachen Mitteln nicht lösbar?

	$a_1$	$q$	$n$	$a_n$	$s_n$
a)	2	$\frac{1}{2}$	6	$\frac{1}{16}$	$3\frac{1}{8}$
b)	3	$\frac{1}{3}$	5	$\frac{1}{27}$	$4\frac{1}{27}$
c)	1	3	6	243	364
d)	1	2	8	128	255
e)	12	$\frac{1}{2}$	10	$\frac{3}{128}$	$23\frac{1}{256}$
f)	6	$\frac{1}{3}$	9	$\frac{2}{187}$	$8\frac{2}{187}$
g)	4	$-\frac{1}{2}$	8	$-\frac{1}{32}$	$\frac{3}{2}$
h)	3	$-\frac{1}{3}$	6	$-\frac{1}{81}$	$4\frac{1}{81}$

In den folgenden Aufgaben 11 bis 22 sind aus den drei gegebenen die zwei fehlenden Größen zu berechnen:

- |                  |                      |                    |          |
|------------------|----------------------|--------------------|----------|
| 11)              | $a_1 = 1$            | $q = 2$            | $n = 7$  |
| 12)              | $a_1 = 4$            | $q = 3$            | $n = 10$ |
| 13)              | $a_1 = 5$            | $q = 4$            | $n = 9$  |
| 14) <sup>1</sup> | $a_1 = 1000$         | $q = 1,04$         | $n = 30$ |
| 15) <sup>1</sup> | $a_1 = 1000$         | $q = 1,035$        | $n = 40$ |
| 16) <sup>1</sup> | $a_1 = 4000$         | $q = 1,05$         | $n = 40$ |
| 17)              | $a_1 = \frac{1}{4}$  | $q = 4$            | $n = 11$ |
| 18)              | $a_1 = 5$            | $q = -2$           | $n = 8$  |
| 19)              | $a_1 = -\frac{2}{3}$ | $q = -\frac{1}{2}$ | $n = 7$  |
| 20)              | $a_7 = 256$          | $q = 4$            | $n = 7$  |
| 21)              | $a_6 = 3125$         | $q = 2\frac{1}{2}$ | $n = 6$  |
| 22)              | $a_n = 262144$       | $q = -2$           | $n = 19$ |

<sup>1</sup> Bei dieser Aufgabe ist nur  $a_n$  zu berechnen.

- 23) Wir nehmen an, man habe am 1. Januar 1801 einen Franken an Zinseszins gelegt. Bei gleichbleibendem Zinsfuß soll sich das Geld nach je 20 Jahren verdoppelt haben und auch in Zukunft in gleicher Art weiter anwachsen. Auf welche Summe wird das Kapital am 1. Januar 1961 angewachsen sein?
- 24) Man teile die Zahl 1562 so in fünf Summanden, daß jeder nachfolgende Summand das Fünffache des vorhergehenden ist.
- 25) Bei einer geometrischen Folge ist die Differenz der zwei ersten Glieder 12, die Differenz zwischen dem dritten und vierten Glied 300. Wie heißen die ersten vier Glieder der Folge?
- 26) Bei einer geometrischen Folge aus sieben Gliedern beträgt die Summe der ersten drei Glieder 26, die Summe der letzten drei 2106. Wie heißt die Folge?
- 27) Die Zahl 104 [169] ist so in drei Teile zu teilen, daß diese eine geometrische Folge bilden und daß deren drittes Glied um 64 [104] größer ist als das erste.
- 28) Eine geometrische Folge beginnt mit 4 [12] und schließt mit 26244  $[\frac{3}{32}]$ ; die Summe aller Glieder beträgt 39364  $[\frac{765}{32}]$ . Wie viele Glieder hat die Folge, und wie groß ist der Quotient?
- 29) Die Summe dreier Zahlen, die eine geometrische Folge bilden, ist 93, ihr Produkt 3375. Wie heißen die drei Zahlen?
- 30) Drei Zahlen sollen eine arithmetische Folge mit der Differenz  $d = \sqrt{2}$  bilden; ihre Quadrate jedoch sollen eine geometrische Folge darstellen. Welches sind die drei Zahlen?
- 31) Zwischen 1 [2] und 3 [512] sollen 10 [7] Glieder so eingeschaltet werden, daß das Ganze eine geometrische Folge darstellt.
- 32) Zwischen je zwei benachbarte Glieder der Folge 1, 2, 4, 8, 16, 32 ist noch ein Glied einzuschalten, so daß wieder eine geometrische Folge entsteht.
- 33) Zwischen die Zahlen 16 und 81 soll man andere Zahlen einschieben, so daß eine geometrische Folge entsteht, deren Summe 211 ist. Man bestimme den Quotienten  $q$  und die Anzahl der Glieder.
- 34) Bilde eine geometrische Folge aus sieben Gliedern, so daß die Summe der sechs letzten Glieder doppelt so groß ist wie die Summe der sechs ersten, welche  $157\frac{1}{2}$  beträgt.